Fault-Tolerant Compression Filters

by Time-Propagated Measurement Fusion

Hyung Keun Lee[a], Jang Gyu Lee[*b]

[a] School of Electronics, Telecommunication and Computer Engineering, Hankuk Aviation University, Korea, hyknlee@hau.ac.kr
[*] Corresponding author. Phone +82+2+880+7308 Fax +82+2+878+8198
[b] Automatic Control Research Center and School of Electrical Engineering and Computer Science, Seoul National University, Korea, jgl@snu.ac.kr

Abstract

For fault-tolerant real-time filtering, an efficient two-filter architecture is proposed. In the two-filter architecture, a host filter is operated permanently. A compression filter is intermittently initialized and operated during prescribed time intervals. to handle measurements susceptible to soft faults. New compression filters are derived based on time-propagated measurement concept. The derived de-correlated full error-state compression filter and correlated partial error-state compression filter facilitate a number of different ways of ordering out-of-normal-sequence measurements, provide efficient state estimates with consistent error covariance information, and make fault detection and isolation convenient.

Key words: real-time, fault, detection, isolation, information, fusion, compression, filter.

1. Introduction

Sensor faults can be classified into two types: a hard fault where the fault magnitude abruptly increases and a soft fault where the fault magnitude slowly changes. Since it is difficult to detect by merely comparing the residuals of synchronously-sampled raw measurements, distributed filter architecture needs to be utilized for its detection and isolation. The most characteristic methods in this category are the multiple model filter (Bar-Shalom & Fortmann, 1988; Zhang & Li, 1998; Hanlon & Maybeck, 2000), the federated filter (Carlson, 1988), and the measurement-subset parallel filter (Da, 1995) .

In addition to the multiple-filter methods, the two-filter method introduced by Kerr (1980) is also a viable choice. In this method, a Kalman Filter (KF) and a state propagator are initialized periodically by the same information. After the initialization, the KF is updated by all the measurements accepting possible faults and the state propagator only propagates in time avoiding any fault. After a prescribed time interval, the KF and the state propagator are compared to each other.

In the cases where less computation is required or information redundancy is not sufficient for fault detection and isolation, two-filter methods could be more attractive than multiple-filter methods. However, as noticed by Da (1994), the conventional two-filter method has a problem in that the reference information (state propagator) deteriorates in proportion to its propagation time. For efficient fault detection, it is desirable to maintain the accuracy of reference information as accurate as possible by accepting as many fault-free measurements as possible.

To deal with this problem, an efficient two-filter architecture shown in Fig. 1 is proposed. Sensors are classified into two groups where the sensor group 1 is only prone to hard faults and the sensor group 2 is additionally prone to soft faults. After a simple test on hard faults, the measurements from the sensor group 1 are directly processed by a Host Filter (HF) that acts as the reference information. As a result, the reference information does not deteriorate quickly. The measurements from the sensor group 2 are processed by a Compression Filter (CF) during a prescribed time interval so that the effects of soft faults, if any, become apparent. By comparing the HF and CF, fault occurrence in the CF is tested. If the CF passes the test, the HF accepts the CF information for improved state estimates.

The proposed De-correlated Full error-state Compression Filter (DFCF) and Correlated Partial error-state Compression Filter (CPCF) are advantageous since they facilitate a number of different ways of ordering Out Of Sequence Measurements (OOSMs) (Lee, 2000; Lee, 2002; Bar-Shalom, 2002; Zhang et al., 2002), provide efficient state estimates with consistent error covariance information, and make fault detection and isolation convenient.

2. Information Preservation Principles

The following two lemmas provide us an insight on what actions are permitted to the CFs to preserve their measurement information. The basic concepts underlying the two lemmas can be found in the contexts by Jazwinski (1970), Maybeck (1979), Kailath(1980), Bar-Shalom & Fortmann (1988), Roecker & McGillem (1988), Da (1995), Gan & Harris (2001).

Lemma 1: Stacking and Maintenance

Suppose that we are given *a priori* state estimate $\hat{x}^- \in \mathbf{R}^n$, direct measurements $y_1 \in \mathbf{R}^{m1}$ and $y_2 \in \mathbf{R}^{m2}$ of the state $x \in \mathbf{R}^n$ satisfying

$$\hat{x}^- = x - e^-, \quad y_1 = H_1 x + v_1, \quad y_2 = H_2 x + v_2,$$

$$\begin{bmatrix} e^- \\ v_1 \\ v_2 \end{bmatrix} \sim \left(\begin{bmatrix} O_{n\times 1} \\ O_{m1\times 1} \\ O_{m2\times 1} \end{bmatrix}, \begin{bmatrix} M & O & O \\ O & R_1 & O \\ O & O & R_2 \end{bmatrix} \right), \ M > O, R_1 > O, R_2 > O. \quad (1)$$

Note that $e \sim (m,r)$ means that the random vector (R.V.) e is Gaussian-distributed with the mean m and the error covariance r. $O_{a\times b}$ and O denote the zero matrices.

Then, the utilization of y_1 and y_2 in obtaining the optimal *a posteriori* state estimate of x is equivalent to

(i) the utilization of $\bar{z} \in \mathbf{R}^{(m1+m2)}$ as shown in Eq. (2).

$$\bar{z} := \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \bar{H} e^- + \bar{v}, \quad \bar{H} := \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}, \quad \begin{matrix} z_1 := y_1 - H_1 \hat{x}^-, \\ z_2 := y_2 - H_2 \hat{x}^-, \end{matrix}$$

$$\begin{bmatrix} e^- \\ \bar{v} \end{bmatrix} \sim \left(\begin{bmatrix} O_{n\times 1} \\ O_{(m1+m2)\times 1} \end{bmatrix}, \begin{bmatrix} M & O \\ O & \bar{R} \end{bmatrix} \right), \quad \bar{R} := \begin{bmatrix} R_1 & O \\ O & R_2 \end{bmatrix}. \quad (2)$$

(ii) the utilization of $z_1 \in \mathbf{R}^{m1}$ and $z'_2 \in \mathbf{R}^{m2}$ as shown in Eqs. (2), (3) and (4).

Partial Update:

$$K_1 = P' H_1^T R_1^{-1} = M H_1^T (H_1 M H_1^T + R_1)^{-1},$$

$$\hat{x}' = \hat{x}^- + K_1 z_1, \ (P')^{-1} = (M)^{-1} + H_1^T R_1^{-1} H_1. \quad (3)$$

Maintenance:

$$z'_2 := z_2 - H_2 K_1 z_1 = H_2 e' + v_2,$$

$$\begin{bmatrix} e' \\ v_2 \end{bmatrix} \sim \left(\begin{bmatrix} O_{n\times 1} \\ O_{n\times 1} \end{bmatrix}, \begin{bmatrix} P' & O \\ O & R_2 \end{bmatrix} \right), \ e' := \hat{x}' - x. \quad (4)$$

Lemma 2: Scaling, De-correlation, and Compression

Suppose that we are given *a priori* state estimate $\hat{x}^- \in \mathbf{R}^n$ and a direct measurement vector $y \in \mathbf{R}^m$ of the state $x \in \mathbf{R}^n$ such that

$$\hat{x}^- = x - e^-, \quad y = Hx + v,$$

$$\begin{bmatrix} e^- \\ v \end{bmatrix} \sim \left(\begin{bmatrix} O_{n\times 1} \\ O_{m\times 1} \end{bmatrix}, \begin{bmatrix} M & S^T \\ S & R \end{bmatrix} \right), \quad \begin{bmatrix} M & S^T \\ S & R \end{bmatrix} > O. \quad (5)$$

To obtain the optimal $\hat{x}^+$ and its error covariance matrix P, the following equations are usually utilized.

$$K = (MH^T + S^T)(HMH^T + HS^T + SH^T + R)^{-1},$$

$$\hat{x}^+ = \hat{x}^- + Kz, \quad z := y - H\hat{x}^- = He^- + v,$$

$$P = (I - KH)M(I - KH)^T + (I - KH)(KS)^T + (KS)(I - KH)^T + KRK^T. \quad (6)$$

The utilization of y (or z) in obtaining the optimal state estimate of $\hat{x}^+$ as shown in Eq. (6) is equivalent to

(i) the utilization of the scaled measurement $\hat{z} \in \mathbf{R}^m$ as shown in Eq. (7) if $\det(C) \neq 0$.

$$\hat{z} := Cz = \hat{H} e^- + \hat{v}, \hat{H} := CH, C \in \mathbf{R}^{n\times n},$$

$$\begin{bmatrix} e^- \\ \hat{v} \end{bmatrix} \sim \left(\begin{bmatrix} O_{n\times 1} \\ O_{n\times 1} \end{bmatrix}, \begin{bmatrix} M & \hat{S}^T \\ \hat{S} & \hat{R} \end{bmatrix} \right), \ \hat{R} := CRC^T, \ \hat{S} := CS. \quad (7)$$

(ii) the utilization of the de-correlated measurement $\bar{z} \in \mathbf{R}^m$ as shown in Eq. (8).

$$\bar{z} := z = \bar{H} e^- + \bar{v}, \quad \bar{H} := H + SM^{-1},$$

$$\begin{bmatrix} e^- \\ \bar{v} \end{bmatrix} \sim \left(\begin{bmatrix} O_{n\times 1} \\ O_{n\times 1} \end{bmatrix}, \begin{bmatrix} M & O \\ O & \bar{R} \end{bmatrix} \right), \ \bar{R} := R - SM^{-1}S^T. \quad (8)$$

(iii) the utilization of the compressed measurement $\tilde{z} \in \mathbf{R}^n$ as shown in Eq. (9) if $\bar{H}^T \bar{H} > O$.

$$\tilde{z} := (\bar{H}^T \bar{R}^{-1} \bar{H})^{-1} \bar{H}^T \bar{R}^{-1} \bar{z} = \tilde{H} e^- + \tilde{v}, \quad \tilde{H} := I_{n\times n},$$

$$\begin{bmatrix} e^- \\ \tilde{v} \end{bmatrix} \sim \left(\begin{bmatrix} O_{n\times 1} \\ O_{n\times 1} \end{bmatrix}, \begin{bmatrix} M & O \\ O & \bar{R} \end{bmatrix} \right), \ \tilde{R} := (\bar{H}^T \bar{R}^{-1} \bar{H})^{-1}. \quad (9)$$

3. Compression Filters

New compression filters are introduced based on the previous introduced two lemmas. Two types of measurements are considered; raw measurements and equivalent measurements. The equivalent measurements correspond to the processed information including the CF and the OOSM. The system model is given as follows:

$$x_{k+1} = F_{k+1,k} x_k + G_k w_k, \quad y_k = h_k x_k + v_k,$$

$$\begin{bmatrix} w_i \\ v_j \end{bmatrix} \sim \left(O, \begin{bmatrix} q_i & O \\ O & r_j \end{bmatrix} \right), \ i, j, k \in \{0, 1, 2, 3, \cdots\}, \quad (10)$$

where w_k and v_k denote white Gaussian noise terms.

Theorem 1: Time-Propagated Measurement

Suppose that we are given *a priori* state estimate $\hat{x}_k^-$ and a raw indirect measurement z_k of x_k satisfying Eqs. (10), (11a), and (11b).

$$z_k := y_k - h_k \hat{x}_k^- = h_k e_k^- + v_k, \ e_k^- := x_k - \hat{x}_k^-, \quad (11a)$$

$$\begin{bmatrix} e_k^- \\ v_k \end{bmatrix} \sim \left(\begin{bmatrix} O_{n\times 1} \\ O_{m\times 1} \end{bmatrix}, \begin{bmatrix} P_k^- & S_k^T \\ S_k & r_k \end{bmatrix} \right), \quad \begin{bmatrix} P_k^- & S_k^T \\ S_k & r_k \end{bmatrix} > O. \quad (11b)$$

Then, $(\hat{x}_{k+1}^-, P_{k+1}^-)$ by Eqs. (12) and (13) and $(\hat{x}_{k+1}^s, P_{k+1}^s)$ by Eqs.(14)-(15) are the same.

i) Raw Indirect Measurement z_k

Measurement Update:

$$K_k = (P_k^- h_k^T + S_k^T)(h_k P_k^- h_k^T + h_k S_k^T + S_k h_k^T + r_k)^{-1},$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k z_k, \quad P_k^+ = P_k^- - K_k(h_k P_k^- + S_k). \quad (12)$$

Time Propagation:

$$\hat{x}_{k+1}^- = F_{k+1,k}\hat{x}_k^+, \; P_k^- = F_{k+1,k}P_k^+ F_{k+1,k}^T + G_k q_k G_k^T. \quad (13)$$

ii) Time-Propagated Equivalent Measurement $\bar{z}_{k+1}$

Time Propagation:

$$\bar{x}_{k+1} = F_{k+1,k}\hat{x}_k^-, \quad (14a)$$

$$\bar{P}_{k+1} = F_{k+1,k}P_k^-(F_{k+1,k})^T + G_k q_k (G_k)^T, \quad (14b)$$

$$\bar{z}_{k+1} = z_k, \quad \bar{H}_{k+1} := h_k(F_{k+1,k})^{-1}, \quad (14c)$$

$$\bar{R}_{k+1} = r_k + h_k F_{k+1,k}^{-1} G_k q_k G_k^T F_{k+1,k}^{-T} h_k^T, \quad (14d)$$

$$\bar{S}_{k+1} = S_k(F_{k+1,k})^T - h_k F_{k+1,k}^{-1} G_k q_k G_k^T. \quad (14e)$$

Measurement Update:

$$\bar{K}_{k+1} = (\bar{P}_{k+1}\bar{H}_{k+1}^T + \bar{S}_{k+1}^T)\begin{pmatrix}\bar{H}_{k+1}\bar{P}_{k+1}\bar{H}_{k+1}^T + \bar{H}_{k+1}\bar{S}_{k+1}^T \\ +\bar{S}_{k+1}\bar{H}_{k+1}^T + \bar{R}_{k+1}\end{pmatrix}^{-1},$$

$$\hat{x}_{k+1}^s = \bar{x}_{k+1} + \bar{K}_{k+1}\bar{z}_{k+1},$$

$$P_k^s = \bar{P}_{k+1} - \bar{K}_{k+1}(\bar{H}_{k+1}\bar{P}_{k+1} + \bar{S}_{k+1}). \quad (15)$$

where

$$\bar{z}_{k+1} = \bar{H}_{k+1}\bar{e}_{k+1} + \bar{v}_{k+1} \quad (16a)$$

$$\bar{e}_{k+1} := x_{k+1} - \bar{x}_{k+1} = F_{k+1,k}e_k^- + G_k w_k \quad (16b)$$

$$\bar{v}_{k+1} := v_k - h_k(F_k)^{-1}G_k w_k \quad (16c)$$

$$\begin{bmatrix}\bar{e}_{k+1} \\ \bar{v}_{k+1}\end{bmatrix} \sim \left(\begin{bmatrix}O_{n\times 1} \\ O_{m\times 1}\end{bmatrix}, \begin{bmatrix}\bar{P}_{k+1} & \bar{S}_{k+1}^T \\ \bar{S}_{k+1} & \bar{R}_{k+1}\end{bmatrix}\right), \begin{bmatrix}\bar{P}_{k+1} & \bar{S}_{k+1}^T \\ \bar{S}_{k+1} & \bar{R}_{k+1}\end{bmatrix} > O. \quad (16d)$$

Theorem 2: Full Error-State CF

Consider the system satisfying Eq. (10). The HF and CF algorithms in Eqs. (17)-(25) yield the same $(\hat{x}_{k0+N}^+, P_{k0+N}^+)$ as the conventional KF algorithm in Eqs. (12) and (13) for any interval length of N.

HF Initialization:

$$\bar{x}_{k0}^h = \hat{x}_{k0}^-, \qquad \bar{P}_{k0}^h = P_{k0}^-. \quad (17)$$

HF Time Propagation:

$$\bar{x}_{k+1}^h = F_{k+1,k}\bar{x}_k^h, \; \bar{P}_{k+1}^h = F_{k+1,k}\bar{P}_k^h F_{k+1,k}^T + G_k q_k (G_k)^T. \quad (18)$$

CF Initialization:

$$Z_{k0}^c = y_{k0} - h_{k0}\bar{x}_{k0}^h, \qquad H_{k0}^c = h_{k0}, \qquad R_{k0}^c = r_{k0}. \quad (19)$$

CF Time Propagation:

$$\hat{Z}_{k+1}^c = Z_k^c, \; \hat{H}_{k+1}^c = H_k^c F_{k+1,k}^{-1},$$

$$\hat{R}_{k+1}^c = R_k^c + (\hat{H}_k^c G_k) q_k (\hat{H}_k^c G_k)^T. \quad (20)$$

CF De-correlation:

$$\bar{Z}_{k+1}^c = \hat{Z}_k^c, \quad \bar{H}_k^c = \hat{H}_k^c\left[I - G_{k-1}q_{k-1}G_{k-1}^T(\bar{P}_k^h)^{-1}\right],$$

$$\bar{R}_k^c = \hat{R}_k^c - (\hat{H}_k^c G_{k-1}q_{k-1}G_{k-1}^T)(\bar{P}_k^h)^{-1}(\hat{H}_k^c G_{k-1}q_{k-1}G_{k-1}^T)^T. \quad (21)$$

New Indirect Measurement Computation:

$$z_k = y_k - h_k\bar{x}_k^h. \quad (22)$$

Choice 1 for z_k: Before $(H_k^c)^T(H_k^c) > O$:

$$Z_k^c = \begin{bmatrix} z_k \\ \bar{Z}_k^c \end{bmatrix}, \quad H_k^c = \begin{bmatrix} h_k \\ \bar{H}_k^c \end{bmatrix}, \quad R_k^c = \begin{bmatrix} r_k & O \\ O & \bar{R}_k^c \end{bmatrix}. \quad (23)$$

Choice 2 for z_k: After $(H_k^c)^T(H_k^c) > O$:

$$R_k^c = \left[(\bar{H}_k^c)^T(\bar{R}_k^c)^{-1}\bar{H}_k^c + h_k^T r_k^{-1} h_k\right]^{-1},$$

$$Z_k^c = R_k^c\left[(\bar{H}_k^c)^T(\bar{R}_k^c)^{-1}\bar{Z}_k^c + h_k^T r_k^{-1} z_k\right], \quad H_k^c = I_{n\times n}. \quad (24)$$

HF update by CF:

$$K_{k0+N}^c = \bar{P}_{k0+N}^h(H_{k0+N}^c)^T\left[H_{k0+N}^c\bar{P}_{k0+N}^h(H_{k0+N}^c)^T + R_{k0+N}^c\right]^{-1},$$

$$\hat{x}_{k0+N}^+ = \bar{x}_{k0+N}^h + K_{k0+N}^c Z_{k0+N}^c,$$

$$P_{k0+N}^+ = \bar{P}_{k0+N}^h - K_{k0+N}^c H_{k0+N}^c \bar{P}_{k0+N}^h. \quad (25)$$

From now on, it will be assumed that the measurement vector arriving at each time step is decomposed as one of

$$(i)\, y_k = \left[(y_k^c)^T \; (y_k^h)^T\right]^T, (ii)\, y_k = y_k^c, \text{ or } (iii)\, y_k = y_k^h, \quad (26)$$

where

$y_k^c = h_k^c x_k + v_k^c$: CF-handled measurement,

$y_k^h = h_k^h x_k + v_k^h$: HF-handled measurement,

$$\begin{bmatrix} v_k^c \\ v_k^h \end{bmatrix} \sim (O_{(m1+m2)\times 1}, r_k), \; r_k = \begin{bmatrix} r_k^c & O_{m1\times m2} \\ O_{m2\times m1} & r_k^h \end{bmatrix}. \quad (27)$$

As compared to the CF in Theorem 2, the DFCF in Theorem 3 permits occasional updating of HF by y_k^h shown in Eqs. (27) and (28). The HF update by the DFCF is the same as shown in Eq. (26)

Theorem 3: De-correlated full error-state CF

Consider the system satisfying Eqs. (10), (26), and (27). The HF and CF algorithms in Eqs. (17), (28)-(38), and (25) yields the same $\hat{x}_{k0+N}^+ \sim (x_{k0+N}, P_{k0+N}^+)$ as the conventional KF algorithm in Eqs. (12) and (13) for any N.

HF Time Propagation:

$$\hat{x}_{k+1}^h = F_{k+1,k}\bar{x}_k^h, \; \hat{P}_{k+1}^h = F_{k+1,k}\bar{P}_k^h F_{k+1,k}^T + G_k q_k (G_k)^T. \quad (28)$$

HF Measurement Update by Raw Measurement:

$$z_k^h = y_k^h - h_k^h \hat{x}_k^h, \quad K_k^h = \hat{P}_k^h (h_k^h)^T \left[h_k^h \hat{P}_k^h (h_k^h)^T + r_k^h \right]^{-1},$$

$$\bar{x}_k^h = \hat{x}_k^h + K_k^h z_k^h, \; \bar{P}_k^h = \hat{P}_k^h - K_k^h h_k^h \hat{P}_k^h. \tag{29}$$

CF Initialization:

$$Z_{k0}^c = y_{k0}^c - h_{k0}^c \bar{x}_{k0}^h, \qquad H_{k0}^c = h_{k0}^c, \qquad R_{k0}^c = r_{k0}^c. \tag{30}$$

CF Time Propagation:

$$\hat{Z}_{k+1}^c = Z_k^c, \; \hat{H}_{k+1}^c = H_k^c F_{k+1,k}^{-1},$$
$$\hat{R}_{k+1}^c = R_k^c + (\hat{H}_k^c G_k) q_k (\hat{H}_k^c G_k)^T. \tag{31}$$

CF De-correlation:

$$\bar{Z}_k^c = \hat{Z}_k^c, \quad \bar{H}_k^c = \hat{H}_k^c \left[I - G_{k-1} q_{k-1} G_{k-1}^T (\bar{P}_k^h)^{-1} \right],$$

$$\bar{R}_k^c = \hat{R}_k^c - (\hat{H}_k^c G_{k-1} q_{k-1} G_{k-1}^T)(\bar{P}_k^h)^{-1} (\hat{H}_k^c G_{k-1} q_{k-1} G_{k-1}^T)^T. \tag{32}$$

CF Maintenance only after HF Update:

$$\bar{Z}_k^c := \bar{Z}_k^c - \bar{H}_k^c K_k^h z_k^h. \tag{33}$$

New Indirect Measurement Computation:

$$z_k^c = y_k^c - h_k^c \bar{x}_k^h \tag{34}$$

Choice 1 for z_k^c: Before $(\bar{H}_k^c)^T (\bar{H}_k^c) > O$:

$$Z_k^c = \begin{bmatrix} z_k^c \\ \bar{Z}_k^c \end{bmatrix}, \quad H_k^c = \begin{bmatrix} h_k^c \\ \bar{H}_k^c \end{bmatrix}, \quad R_k^c = \begin{bmatrix} r_k^c & O \\ O & \bar{R}_k^c \end{bmatrix}. \tag{35}$$

Choice 2 for z_k^c: After $(\bar{H}_k^c)^T (\bar{H}_k^c) > O$:

$$R_k^c = \left[(\bar{H}_k^c)^T (\bar{R}_k^c)^{-1} \bar{H}_k^c + (h_k^c)^T (r_k^c)^{-1} h_k^c \right]^{-1}, \quad H_k^c = I_{n \times n},$$

$$Z_k^c = R_k^c \left[(\bar{H}_k^c)^T (\bar{R}_k^c)^{-1} \bar{Z}_k^c + (h_k^c)^T (r_k^c)^{-1} z_k^c \right]. \tag{36}$$

To reduce the computational burden of the two CFs appearing in Theorem 2 and 3, we further assume that the measurement $y_k^c \in \mathbf{R}^{m1}$ handled by the CF is related to only a fixed subset of the system states, i.e., there exist a time-invariant mapping $T^1 \in \mathbf{R}^{m1 \times n}$ and a time-varying transformation $\phi_k \in \mathbf{R}^{m1 \times m1}$ such that

$$y_k^c = \phi_k x_k^1 + v_k^c, \qquad x_k^1 = T^1 x_k, \qquad \det(\phi_k) \neq 0. \tag{37}$$

If we define a nonsingular similarity transformation $T \in \mathbf{R}^{n \times n}$ based on the $T^1 \in \mathbf{R}^{m1 \times n}$ and its complement $T^2 \in \mathbf{R}^{(n-m1) \times n}$ such that

$$T := \left[(T^1)^T \; \vdots \; (T^2)^T \right]^T, \quad \det(T) \neq 0, \tag{38}$$

an equivalent system satisfying the following partitioning can be found based on the similarity transformation T and Lemma 2-(ii).

$$h_k^c = h = \left[I_{m1 \times m1} \; \vdots \; O \right],$$

$$x_k = \begin{bmatrix} x_k^1 \\ x_k^2 \end{bmatrix}, \; F_{k,j} = \begin{bmatrix} F_{k,j}^{11} & F_{k,j}^{12} \\ F_{k,j}^{21} & F_{k,j}^{22} \end{bmatrix}, \; G_k = \begin{bmatrix} G_k^1 \\ G_k^2 \end{bmatrix}. \tag{39}$$

According to the state decomposition shown in Eq. (39) and the HF algorithm shown in Eqs. (28) and (29), the HF estimation error propagates in time as follows.

$$z_k^h = h_k^h \hat{e}_k^h + v_k^h, \quad \bar{e}_k^h = \left(I - K_k^h h_k^h \right) \hat{e}_k^h - K_k^h v_k^h,$$

$$\hat{e}_{k+1}^h = F_{k+1,k} \bar{e}_k^h + G_k w_k, \tag{40}$$

where

$$\bar{e}_k^h := x_k - \bar{x}_k^h, \; \bar{e}_k^h \sim \left(O_{n \times n}, \bar{P}_k^h \right), \; \bar{P}_k^h = \begin{bmatrix} \bar{P}_k^{11} & \bar{P}_k^{12} \\ \bar{P}_k^{21} & \bar{P}_k^{22} \end{bmatrix},$$

$$\hat{e}_k^h := x_k - \hat{x}_k^h, \; \hat{e}_k^h \sim \left(O_{n \times n}, \hat{P}_k^h \right), \; \hat{P}_k^h = \begin{bmatrix} \hat{P}_k^{11} & \hat{P}_k^{12} \\ \hat{P}_k^{21} & \hat{P}_k^{22} \end{bmatrix}. \tag{41}$$

Based on the state and measurement decompositions shown in Eqs. (40)-(44), a CF is introduced in Theorem 4.

Theorem 4: Correlated partial error-state CF

Consider the system satisfying Eqs. (10), (27), and (28) and the HF algorithm in Eqs. (18), (29)-(31), and (50). Then, the CF algorithm in Eqs. (45)-(49) yields an efficient compressed measurement that provides at least as much information on x_{k0+N}^c than a single raw measurement.

CF Initialization:

$$Z_{k0}^c = y_{k0}^c - h \bar{x}_{k0}^h, \; R_{k0}^c = r_{k0}^c, \; S_{k0}^c = O_{m1 \times n}. \tag{42}$$

CF Time-Propagation:

$$\bar{Z}_{k+1}^c = F_{k+1,k}^{11} Z_k^c,$$

$$\bar{R}_{k+1}^c = F_{k+1,k}^{11} \bar{R}_k^c (F_{k+1,k}^{11})^T + F_{k+1,k}^{12} \bar{P}_k^{22} (F_{k+1,k}^{12})^T + G_k^1 q_k (G_k^1)^T$$
$$- S_k^2 (F_{k+1,k}^{12})^T - F_{k+1,k}^{12} (S_k^2)^T,$$
$$\bar{S}_{k+1}^c = F_{k+1,k}^{11} S_k^c (F_{k+1,k})^T$$
$$- F_{k+1,k}^{12} \left[\bar{P}_k^{21} \; \vdots \; \bar{P}_k^{22} \right] (F_{k+1,k})^T - G_k^1 q_k (G_k)^T. \tag{43}$$

CF Maintenance only after HF Update:

$$\bar{Z}_k^c := \bar{Z}_k^c - h K_k^h z_k^h, \qquad \bar{S}_k^c := \bar{S}_k^c (I - K_k^h h_k^h)^T. \tag{44}$$

New Indirect Measurement Computation for CF:

$$z_k^c = y_k^c - h_k^c \bar{x}_k^h \tag{45}$$

CF Compression:

$$R_k^c = \left[\left(\bar{R}_k^c \right)^{-1} + (r_k^c)^{-1} \right]^{-1}, \; S_k^c = R_k^c \left(\bar{R}_k^c \right)^{-1} \bar{S}_k^c,$$

$$Z_k^c = R_k^c \left[\left(\bar{R}_k^c \right)^{-1} \bar{Z}_k^c + (r_k^c)^{-1} z_k^c \right]. \tag{46}$$

HF update by CF:

$$K^c_{k0+N} = \left[\bar{P}^h_{k0+N}(H^c_{k0+N})^T + (S^c_{k0+N})^T\right]\begin{bmatrix} H^c_{k0+N}\bar{P}^h_{k0+N}(H^c_{k0+N})^T \\ +H^c_{k0+N}(S^c_{k0+N})^T \\ +S^c_{k0+N}(H^c_{k0+N})^T + R^c_{k0+N} \end{bmatrix}^{-1},$$

$$\hat{x}^+_{k0+N} = \bar{x}^h_{k0+N} + K^c_{k0+N}Z^c_{k0+N},$$

$$P^+_{k0+N} = \bar{P}^h_{k0+N} - K^c_{k0+N}\left[S^c_{k0+N} + H^c_{k0+N}\bar{P}^h_{k0+N}\right]. \quad (47)$$

4. Simulation

To demonstrate how the proposed CFs can be utilized in detecting and isolating soft faults, a Monte-Carlo simulation is performed by the following simplified model.

$$F^{11}_{k+1,k} = 1,\ F^{12}_{k+1,k} = \Delta,\ F^{21}_{k+1,k} = 0,\ F^{22}_{k+1,k} = 1,$$

$$G^1_k = \Delta^2/2,\ G^2_k = \Delta,\ \Delta = 0.2,\ q_k = 25\ (\mathrm{m}^2/\mathrm{sec}^4),$$

$$h^h_k = [0 \quad 1],\ r^h_k = 0.01\ (\mathrm{m}^2/\mathrm{sec}^2),\ k = 0, 1, 2, 3, \cdots,$$

$$h^c_j = [1 \quad 0],\ r^c_k = 100\ (\mathrm{m}^2),\ j = 0, 5, 10, 15, \cdots. \quad (48)$$

To simulate a soft fault, the following ramp-type error is added to the position measurements after 700 seconds.

$$\mu_{j+5} = \mu_j + 0.1,\ \mu_{j0} = 0. \quad (49)$$

In the simulation, (a) the conventional KF algorithm, (b) the DFCF/HF pair, and (c) the CPCF/HF pair are compared. The HFs are updated at every 0.2 seconds by the velocity measurements and the CFs are updated at every 1 seconds by the position measurements. The HF updates by the CF are performed at every 30 seconds. Before the KF updates and the HF updates by the CF, a test statistic ζ_k is computed for fault detection as follows.

$$\zeta_k = \begin{cases} (z^c_k)^T\left[h^c_k P^-_k (h^c_k)^T + r^c_k\right]^{-1} z^c_k & : \text{measurement} \\ (Z^c_k)^T\begin{bmatrix} H^c_k\bar{P}^h_k(H^c_k)^T + S^c_k(H^c_k)^T \\ +H^c_k(S^c_k)^T + R^c_k \end{bmatrix}^{-1} Z^c_k & : \text{CF} \end{cases} \quad (50)$$

If there is no fault, z^c_k and Z^c_k in Eq. (50) is Gaussian-distributed and ζ_k is $\chi^2(0, DOF)$ -distributed where DOF indicates the degree of freedom corresponding to the dimension of z^c_k or Z^c_k in Eq. (50). Based on the test statistic ζ_k, the following hypothesis test (Bar-Shalom & Fortmann, 1988) is performed.

$$H_0: \zeta_k < \varepsilon \text{ (no fault)},\quad H_1: \zeta_k \geq \varepsilon \text{ (fault)} \quad (51)$$

where ε is determined by the false alarm probability P_{FA} and DOF. Given $P_{FA} = 0.05$, $\varepsilon = 3.84$ for the KF and the CPCF since $DOF = 1$ and $\varepsilon = 7.38$ for the DFCF since $DOF = 2$.

Fig. 2 shows the simulation result where the injected fault, the actual one-sigma values, and the filter-estimates of one-sigma values are plotted. To get actual one-sigma values, total 360 ensembles are run for each method. After 700 seconds, the actual one-sigma value of the KF accumulates largely. However, the one-sigma value based on the KF's error covariance matrix does not increase which means that the fault-affected measurements harm the KF. During the same period, the DFCF/HF and CPCF/HF pairs show different characteristics. They show slight increase in the one-sigma values both by the actual error statistics and by the filter's error covariance matrix, which means that they rejected to use the fault-affected measurements.

5. Conclusion

For fault-tolerant real-time estimation, a two-filter architecture is proposed. The architecture consists of a host filter and a compression filter. The host filter performs conventional Kalman filter algorithm treating the final state estimate of the compression filter as a usual measurement. The compression filter handles out-of-normal-sequence measurements to protect the host filter from possible soft faults. A simulation result evaluated the efficiency of the compression filters.

ACKNOWLEDGEMENTS

This work has been supported by the Automatic Control Research Center (ACRC) and the BK21 Research Division for Information Technology, Seoul National University, and by the Agency for Defense Development (ADD).

REFERENCES

Bar-Shalom, Y. (2002). Update with out-of-sequence measurements in tracking: exact solution, *IEEE Tr. on Aerospace and Electronic Systems*, 38(3), 769-778
Bar-Shalom, Y., & Fortmann, T. E. (1988), *Tracking and Data Association*, Academic Press.
Carlson, N. A. (1988). Federated filter for fault-tolerant integrated navigation systems, *Proceeding of IEEE Position, Location, and Navigation Symposium*, 110-119.
Da, R. (1994). Failure detection of dynamical systems with state chi-square test, *AIAA Journal of Guidance, Control, and Dynamics*, 17(2), 271-277.
Da, R., & and Lin, C. F. (1995). A new failure detection approach and its applications to GPS autonomous integrity monitoring, *IEEE Tr. on Aerospace and Electronic Systems*, 31(1), 499-506.
Gan, Q., & Harris, C. J. (2001). Comparison of two measurement fusion methods for Kalman-filter-based multisensor data fusion, *IEEE Tr. on Aerospace and Electronic Systems*, 37(1), 273-279
Hanlon, P. D., & Maybeck, P. D. (2000). Multiple-model adaptive estimation using a residual correlation Kalman filter bank, *IEEE Tr. on Aerospace and Electronic Systems*, 36(2), 803-805.
Jazwinski, A. H. (1970). *Stochastic Processes and Filtering Theory*, Academic Press.
Kailath, T. (1980). *Linear Systems*, Prentice Hall
Kerr, T. H. (1980). Statistical analysis of a two-ellipsoid overlap test for real-time failure detection, *IEEE Tr. on Aerospace and Electronic Systems*, 25(4), 762-772.
Lee, H. K., Lee., J. G., Jee, G. I., & Park, C. G. (2000). Measurement maintenance and compression for real-time Kalman filter, *Proceedings of the 3rd Asian Control Conference*, Shanghai, 1440-1445
Lee, H. K. (2002). *Time-Propagated Measurement Fusion and Its Application to GPS Multipath Detection and Isolation,* Ph.D. thesis, Seoul National University, Korea

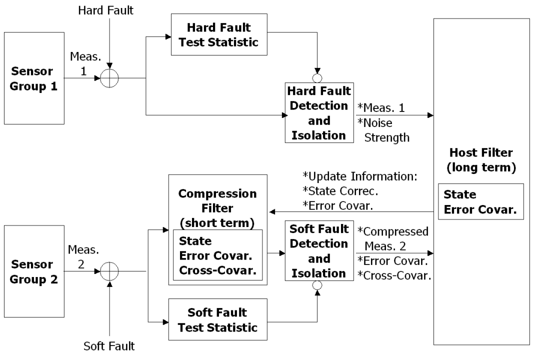

Fig. 1 Two-filter architecture against soft faults

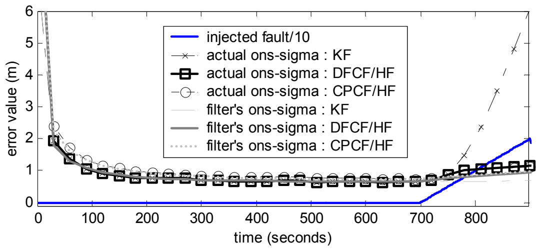

Fig. 2 Performances of Kalman filter, de-correlated full error-state compression filter/host filter pair, and correlated partial error-sate compression filter/host filter pair

Maybeck, P. S. (1979). *Stochastic Models, Estimation and Control*, Vol. 1, Academic Press

Roecker, J. A., & McGillem, C. D. (1988). Comparison of two-sensor tracking methods based on state vector fusion and measurement fusion, *IEEE Tr. on Aerospace and Electronic Systems*, 24(4), 447-449

Zhang, K., Li, X. R., & Zhu, Y. (2002). Optimal update with out-of-sequence measurements for distributed filtering, Proceedings of the 5th International Conf. Information Fusion, Annapolis, 1519-1526

Zhang, Y., & Li, X. R. (1998). Detection and diagnosis of sensor and actuator failures using IMM estimator, *IEEE Tr. on Aerospace and Electronic Systems*, 34, 1293-1313

Appendix A. Proof of Theorem 1

If we do not update $\hat{x}_k^-$ by z_k and propagate it by Eq. (14a), Eq. (16b) is obtained due to Eqs. (10) and (11a). Substituting Eq. (16b) to Eq. (11a) and defining $\overline{z}_{k+1} := z_k$, Eqs. (14c)-(14e), (16a), (16c), and (16d) are obtained based on Eqs. (10), (16b), and the independence between w_k, v_k, and e_v^-. Eq. (14b) is obtained by Eqs. (16b) and (16d). Combining Eqs. (14b)-(14f), it can be verified that

$$\overline{K}_{k+1} = F_{k+1,k}K_k\,,\ \overline{P}_{k+1}\overline{H}_{k+1}^T + \overline{S}_{k+1}^T = F_{k+1,k}(P_k^- h_k^T + S_k^T)\,,$$

$$\begin{aligned}\overline{H}_{k+1}\overline{P}_{k+1}\overline{H}_{k+1}^T + \overline{H}_{k+1}\overline{S}_{k+1}^T + \overline{S}_{k+1}\overline{H}_{k+1}^T + \overline{R}_{k+1} \\ = h_k P_k^- h_k^T + h_k S_k^T + S_k h_k^T + r_k.\end{aligned} \quad \text{(A1)}$$

Substituting Eq. (A1) to Eq. (15) and comparing its results with Eq. (13), it can be verified that

$$\hat{x}_{k+1}^s = \hat{x}_{k+1}^-\,, \quad P_{k+1}^s = P_{k+1}^-\,. \quad \text{(A2)}$$

Appendix B. Proof of Theorem 2

Since Eqs. (17) and (19) correspond to making an indirect measurement from a direct measurement, no information is lost. After the HF and CF time propagation based on Eqs. (18) and (20), the equivalence is maintained by Theorem 1 with $\widehat{S}_{k+1}^c = -\widehat{H}_{k+1}^c G_k q_k G_k^T$. After the CF de-correlation by Eq. (21), the equivalence is maintained by Lemma 2-(ii) and the following equation.

$$\widehat{S}_k^c = -\widehat{H}_k^c G_{k-1} q_{k-1} G_{k-1}^T\,, \quad \overline{S}_k^c = O \quad \text{(B1)}$$

Successive application of the indirect measurement computation and the CF augmentation before the full state observability $(H_k^c)^T(H_k^c) > O$ preserves the equivalence by Lemma 1-(i). Alternatively, the successive application of the indirect measurement computation and the CF compression based on Eqs. (23) and (25) also preserves the equivalence by Lemma 2-(iii).

Appendix C. Proof of Theorem 3

The differences between Eqs. (28)-(36) and Eqs. (18)-(25) occur only in Eqs. (29) and (33). Eq. (29) represents the processing of the measurement stream $\{y_k^h\}$ by the optimal KF algorithm. Thus, no information is lost by Eq. (29). The CF maintenance in Eq. (33) should be performed whenever the HF measurement update in Eq. (29) occurs. By Lemma 1-(ii), no information is lost by Eq. (33).

Appendix D. Proof of Theorem 4

Assume that Z_k^c is given as follows.

$$Z_k^c = h\overline{e}_k^h + v_k^c = \overline{e}_k^1 + V_k^c\,,\ R_k^c = E[(V_k^c)(V_k^c)^T]\,,$$

$$S_k^c = E[(V_k^c)(\overline{e}_k^h)^T] = \left[S_k^1 \;\vdots\; S_k^2\right]. \quad \text{(D1)}$$

Between the k-th and $(k+1)$-th time steps, Z_k^c is time-propagated, scaled, and maintained to $\overline{Z}_{k+1}^c$ as follows.

$$\overline{Z}_{k+1}^c = F_{k+1,k}^{11}Z_k^c - hK_k^h z_k^h = \overline{e}_{k+1}^1 + \overline{V}_{k+1}^c\,,$$

$$\overline{V}_{k+1}^c = F_{k+1,k}^{11}V_k^c - F_{k+1,k}^{12}\overline{e}_k^2 - G_k^1 w_k\,. \quad \text{(D2)}$$

By Theorem 1, Lemma 2-(i), Lemma 1-(ii), and the independence of w_k from V_k^c and $\overline{e}_k^h$, $\overline{R}_{k+1}^c$ in Eq. (43) and $\overline{S}_{k+1}^c$ (44) are obtained based on the following definitions.

$$\overline{R}_{k+1}^c := E[(\overline{V}_{k+1}^c)(\overline{V}_{k+1}^c)^T]\,,\ \overline{S}_{k+1}^c = E[(\overline{V}_{k+1}^c)(\overline{e}_{k+1}^h)^T]\,. \quad \text{(D3)}$$

As soon as the new measurement z_{k+1}^c arrives, more information on $\overline{e}_{k+1}^1$ is available to the CF as follows.

$$z_{k+1}^c = \overline{e}_{k+1}^1 + v_{k+1}^c\,,\ \begin{bmatrix} v_{k+1}^c \\ \overline{V}_{k+1}^c \end{bmatrix} \sim \left(O, \begin{bmatrix} r_{k+1}^c & O \\ O & \overline{R}_{k+1}^c \end{bmatrix}\right). \quad \text{(D4)}$$

Given Eqs. (D2) and (D4), Eq. (46) yields the equivalent measurement with more information on $\overline{e}_{k+1}^1$ than the original measurement z_{k+1}^c.