항법 및 측위 수신기 신호처리의 기초

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Basics

Sine/Cosine 관련 법칙

Sine/Cosine 법칙

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

Euler 법칙

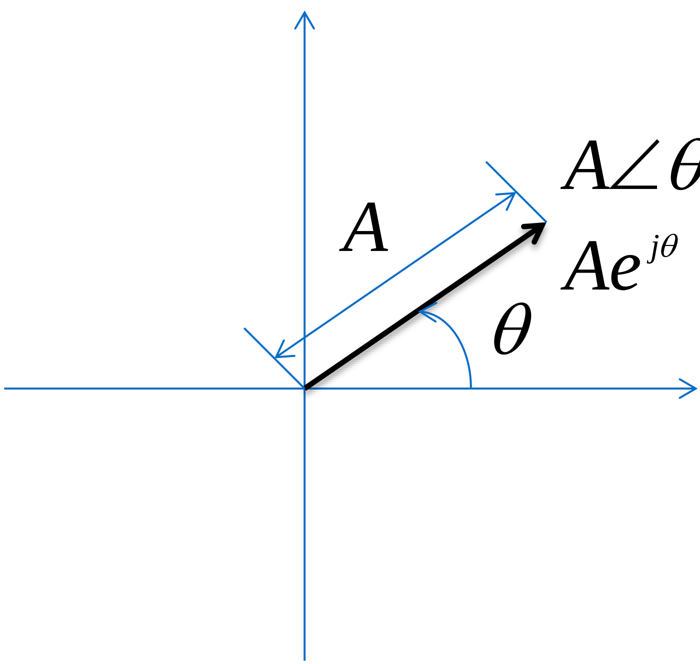
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

복소수와 삼각함수 사이의 관계

$$\sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{j-\theta}\right), \qquad \cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{j-\theta}\right)$$

Time Domain Signal vs Phasor

Assumption : Only known, fixed, and single frequency component exists



$$A\angle\theta = A\cos(2\pi ft + \theta)$$

$$Ae^{j\theta} = A\cos(2\pi ft + \theta) + jA\sin(2\pi ft + \theta)$$

Auto/Cross Correlation

Auto-correlation on infinite-horizon (aperiodic)

$$R_x(t) = \int_{-\infty}^{\infty} x(\tau)x^*(\tau - t)d\tau$$

Cross-correlation on infinite-horizon (aperiodic)

$$R_{yx}(t) = \int_{-\infty}^{\infty} y(\tau)x^*(\tau - t)d\tau$$

Auto-correlation on finite-horizon (periodic)

$$R_x^T(t) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(\tau)x^*(\tau - t)d\tau$$

Cross-correlation on finite-horizon (periodic)

$$R_{yx}^T(t) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T} y(\tau)x^*(\tau - t)d\tau$$

Sine/Cosine Formulae for Correlations

$$\left\langle T = \frac{1}{mf} \right\rangle$$

$$\frac{1}{T}\int_{-T/2}^{T/2} \sin(2\pi mft)\sin(2\pi mft)\,d\tau$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \sin^2(2\pi mft)\,d\tau = \frac{1}{T}\int_{-T/2}^{T/2} \frac{1-\cos(4\pi mft)}{2}\,d\tau = \frac{1}{2}$$

$$\frac{1}{T}\int_{-T/2}^{T/2} \cos(2\pi mft)\cos(2\pi mft)\,d\tau$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \cos^2(2\pi mft)\,d\tau = \frac{1}{T}\int_{-T/2}^{T/2} \frac{1+\cos(4\pi mft)}{2}\,d\tau = \frac{1}{2}$$

$$\frac{1}{T}\int_{-T/2}^{T/2} \sin(2\pi mft)\cos(2\pi mft)\,d\tau$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \frac{\sin(4\pi mft)}{2}\,d\tau = 0$$

$$\left\langle T = \frac{1}{pf},\ p = \text{least common multiple of } m \text{ and } n \right\rangle$$

$$\frac{1}{T}\int_{-T/2}^{T/2} \sin(2\pi mft)\sin(2\pi nft)\,d\tau = 0$$

$$\frac{1}{T}\int_{-T/2}^{T/2} \cos(2\pi mft)\cos(2\pi nft)\,d\tau = 0$$

$$\frac{1}{T}\int_{-T/2}^{T/2} \sin(2\pi mft)\cos(2\pi nft)\,d\tau = 0$$

Fourier Series with Real Coefficients

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(2\pi n f_1 t\right) + \sum_{n=1}^{\infty} b_n \sin\left(2\pi n f_1 t\right)$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos\left(2\pi n f_1 t\right)dt \qquad n = 0,1,2,\ldots$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin\left(2\pi n f_1 t\right)dt \qquad n = 0,1,2,\ldots$$

Fourier Series with Complex Coefficients

$$x(t) = \frac{C_0}{2} + \sum_{n=1}^{\infty} C_n \cos\left(2\pi n f_1 t - \theta_n\right) \qquad n = 1, 2, 3, \ldots$$

where: $C_n = \sqrt{a_n^{\ 2} + b_n^{\ 2}}$ and $\theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$

Fourier Transform

$$X(f) = F\left[x(t)\right] = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

$$x(t) = F^{-1}\left[X(f)\right] = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

$$\langle \omega = 2\pi f \rangle$$

Laplace Transform

$$X(s) = L\left[x(t)\right] = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

$$x(t) = L^{-1}\left[X(s)\right] = \int_{-\infty}^{\infty} X(s)e^{st}ds$$

- Laplace transform is applicable to any signal
- Fourier transform is applicable only to periodic signal

Parseval's Theorem

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

Energy/Power in Time Domain

Energy In Time Domain

$$E = \int_{-\infty}^{\infty} \left| x(t) \right|^2 dt$$

(Average) Power In Time Domain

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left| x(t) \right|^2 dt$$

Energy/Power Spectral Density in Frequency Domain

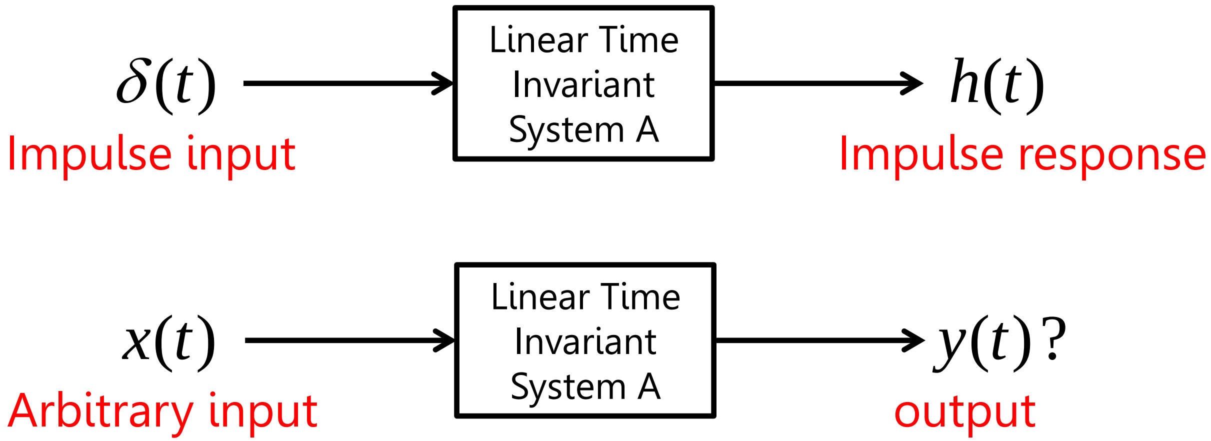
Energy Spectral Density

$$ESD_x(f) = F[\int_{-\infty}^{\infty} x(\tau)x^*(\tau - t)d\tau] = F[R_x(t)]$$

Power Spectral Density

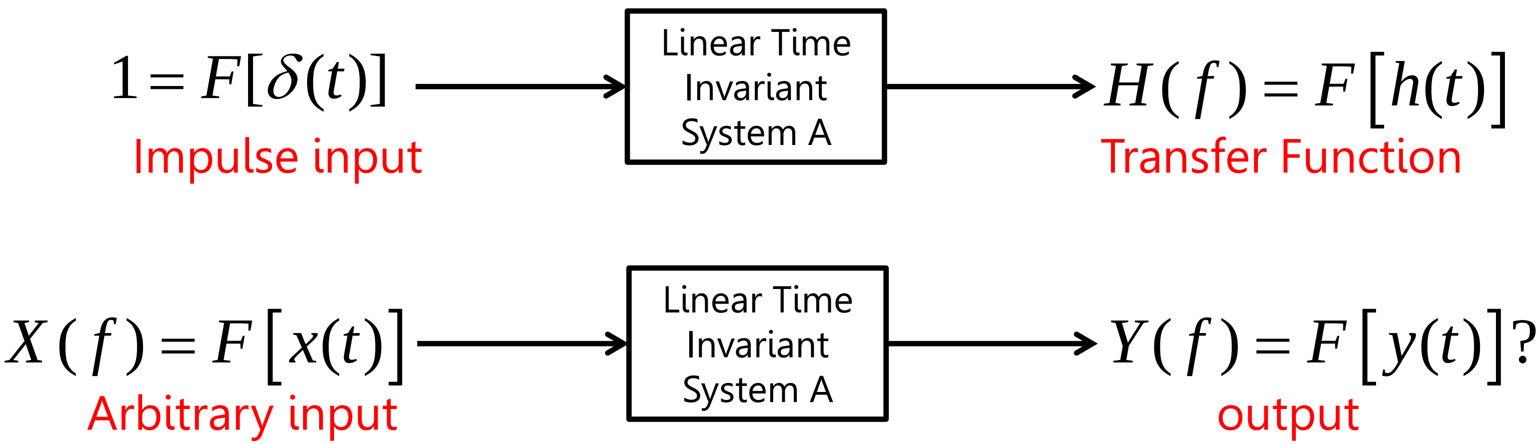
$$PSD_x(f) = F[\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(\tau)x^*(\tau - t)d\tau] = F[R_x^T(t)]$$

Time Domain Response by Convolution Integral



$$y(t) = h(t) \circ x(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

Frequency Domain Response by Transfer Function



$$Y(f) = H(f)X(f)$$

Input/Output Cross-Correlation and Transfer Function

$$R_{xy}^{T}(t) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(\tau)y^*(\tau - t)d\tau$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(\tau)\int_{-\infty}^{\infty} x^*(\tau - t - \alpha)h^*(\alpha)d\alpha d\tau$$

$$= \int_{-\infty}^{\infty} h^*(\alpha)\left[\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(\tau)x^*(\tau - t - \alpha)d\tau\right]d\alpha$$

$$= \int_{-\infty}^{\infty} h^*(\alpha)R_{x}^{T}(t+\alpha)d\alpha = \int_{-\infty}^{\infty} h^*(\tau)R_{x}^{T}[\tau - (-t)]d\alpha$$

$$= h^*(-t)\circ R_{x}^{T}(-t)$$

In summary,

$$R_{xy}^{T}(t) = h^*(-t)\circ R_{x}^{T}(-t) \quad \Longrightarrow \quad \boxed{R_{xy}^{T}(-t) = h^*(t)\circ R_{x}^{T}(t)}$$

Output Auto-Correlation and Transfer Function

$$R_y^T(t) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} y(\tau) y^*(\tau - t) d\tau$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} y^*(\tau - t) \int_{-\infty}^{\infty} x(\tau - \sigma) h(\sigma) d\sigma d\tau$$

$$= \int_{-\infty}^{\infty} h(\sigma) \left[\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(\tau - \sigma) y^*(\tau - t) d\tau \right] d\sigma$$

$$= \int_{-\infty}^{\infty} h(\sigma) R_{xy}^T(t - \sigma) d\sigma = \int_{-\infty}^{\infty} h(\sigma) R_{xy}^T[\sigma - (-t)] d\sigma$$

$$= h(t) \circ R_{xy}^T(-t) = h(t) \circ h^*(t) \circ R_x^T(t)$$

In summary,

$$R_y^T(t) = h(t) \circ h^*(t) \circ R_x^T(t)$$

Input/Output Power Spectral Density and Transfer Function

$$PSD_y(f) = F\left[R_y^T(t)\right] = F\left[h(t) \circ h^*(t) \circ R_x^T(t)\right]$$

$$= F\left[h(t)\right] F\left[h^*(t)\right] F\left[R_x^T(t)\right]$$

$$= H(f) H^*(f) PSD_x(f)$$

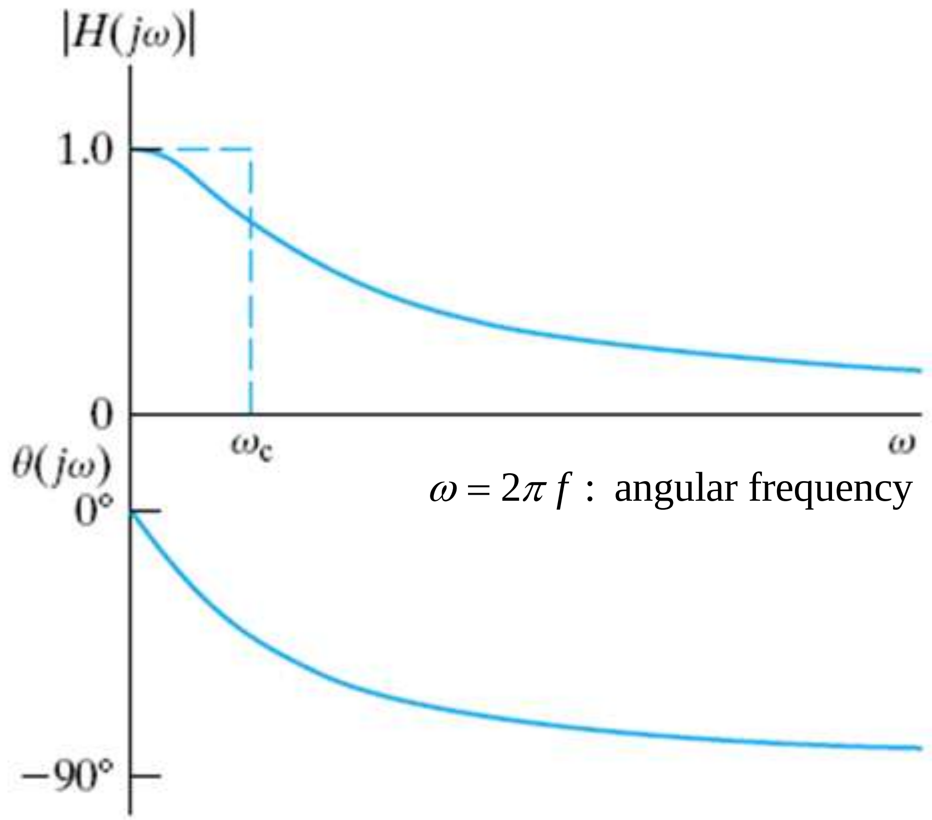
$$= \left|H(f)\right|^2 PSD_x(f)$$

In summary,

$$PSD_y(f) = \left|H(f)\right|^2 PSD_x(f)$$

Frequency Response : Bode Plot of Transfer Function

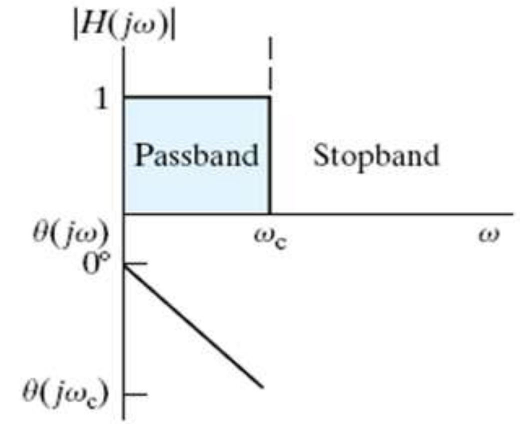
Magnitude Plot



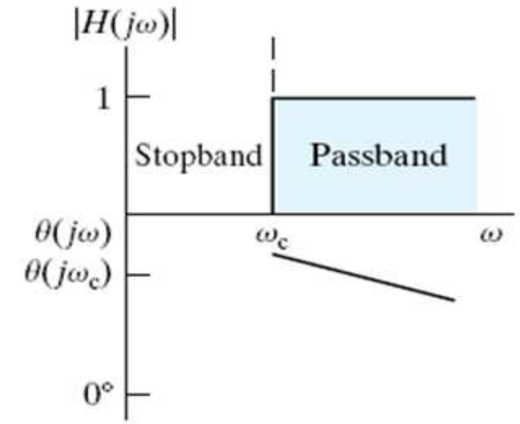
$\omega = 2\pi f$: angular frequency

Phase Plot

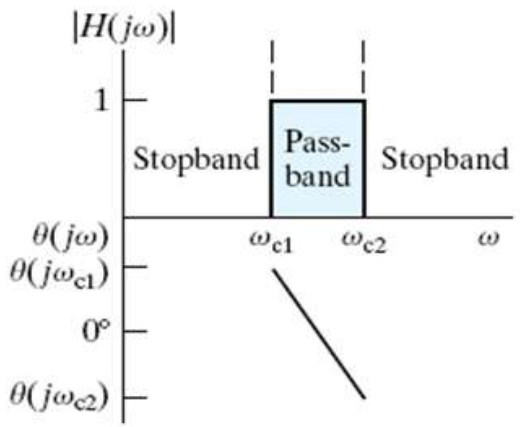
Frequency Response Examples



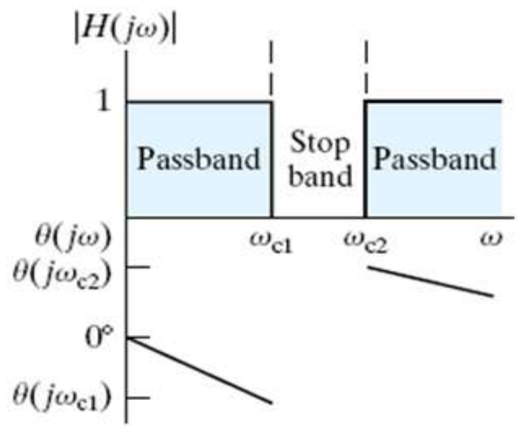
low-pass filter



high-pass filter

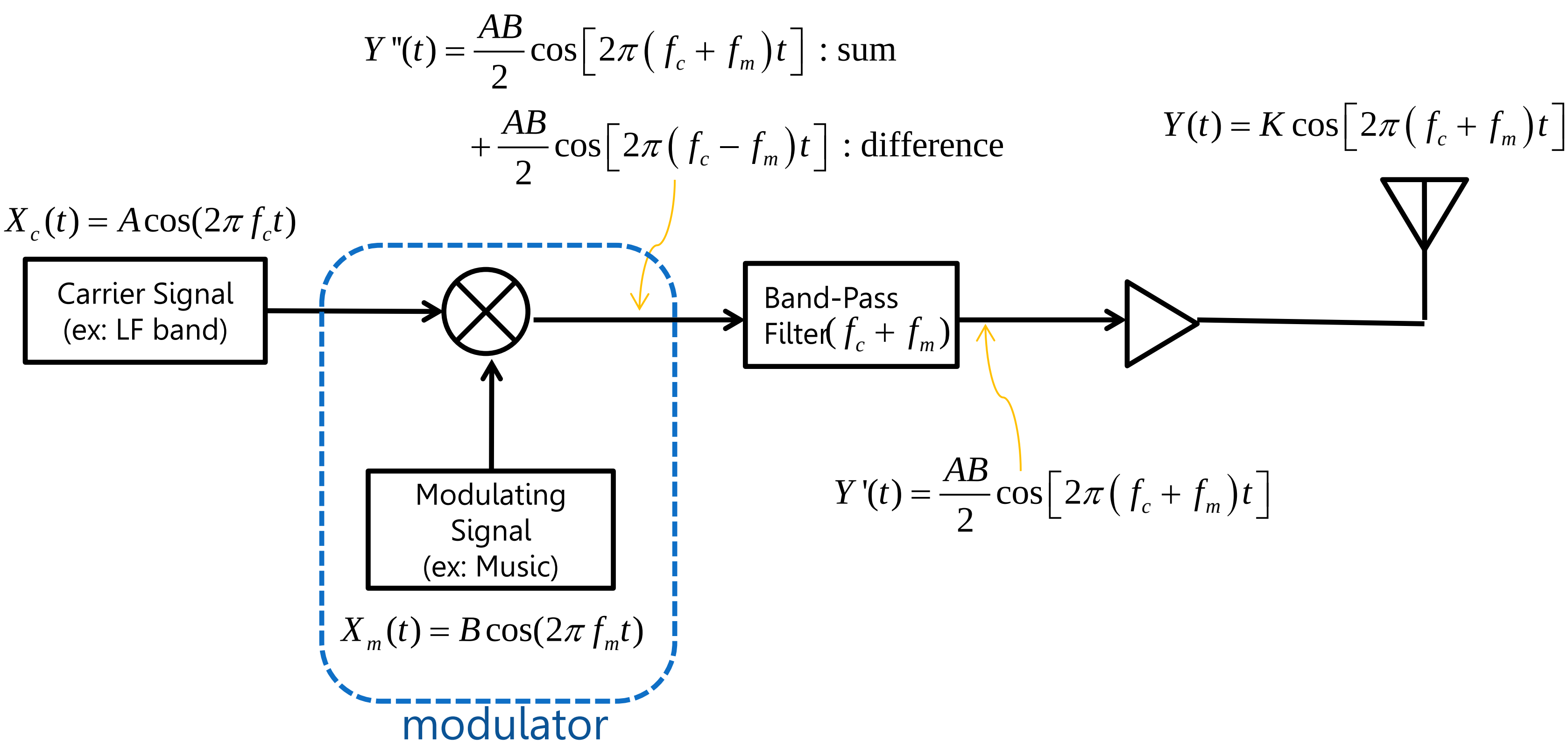


band-pass filter

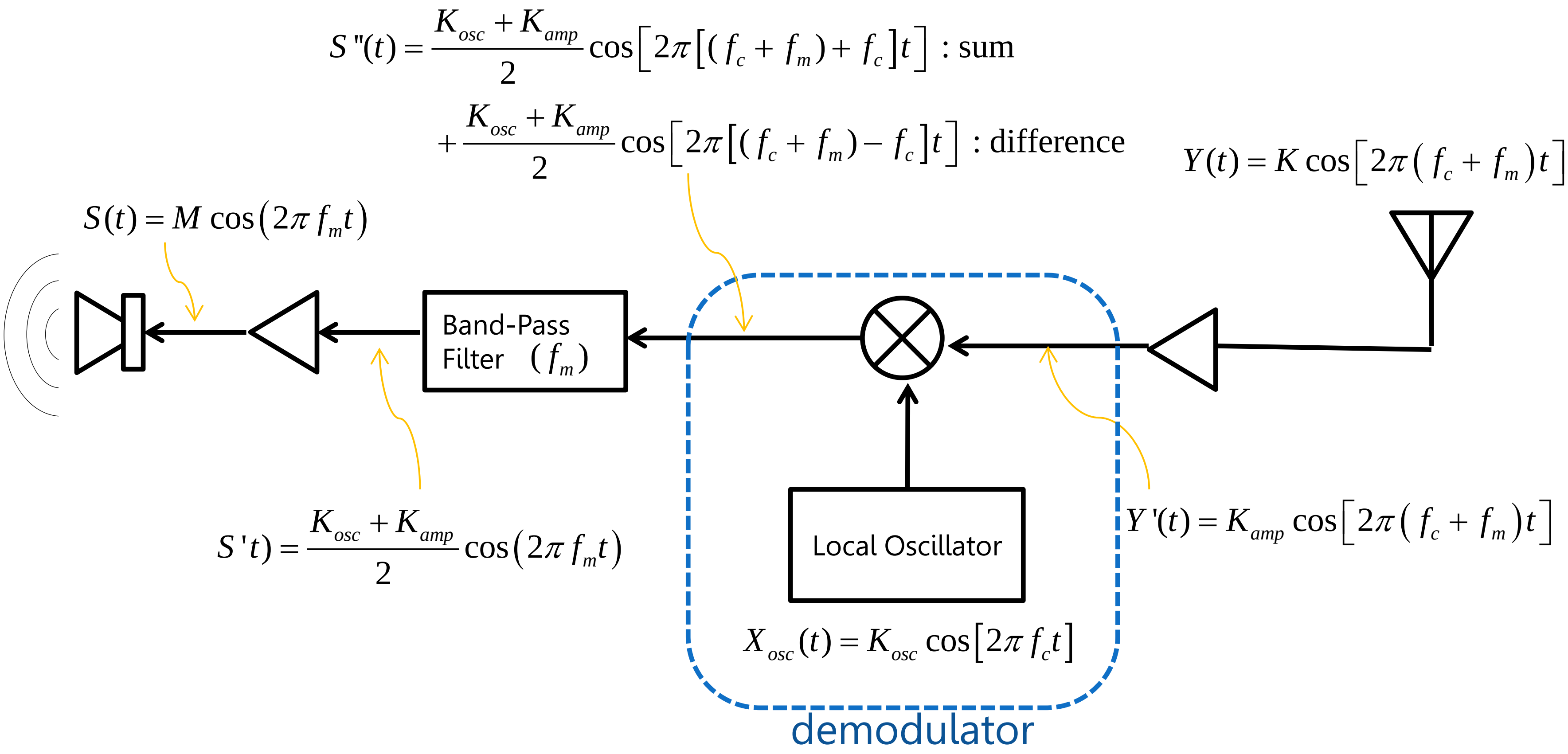


band-reject filter

Modulation (Transmitter)



Demodulation（Receiver）



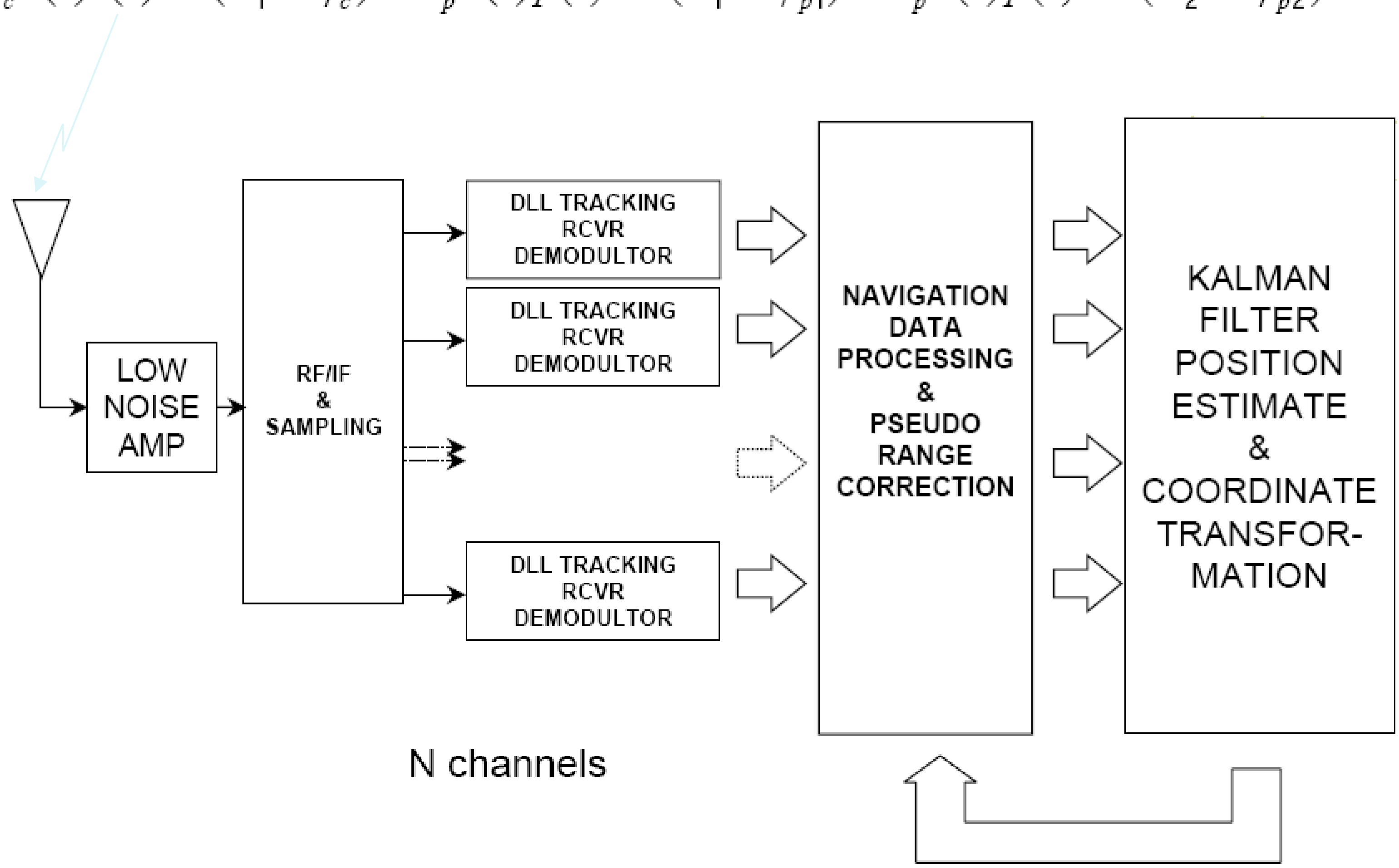
GNSS Receiver

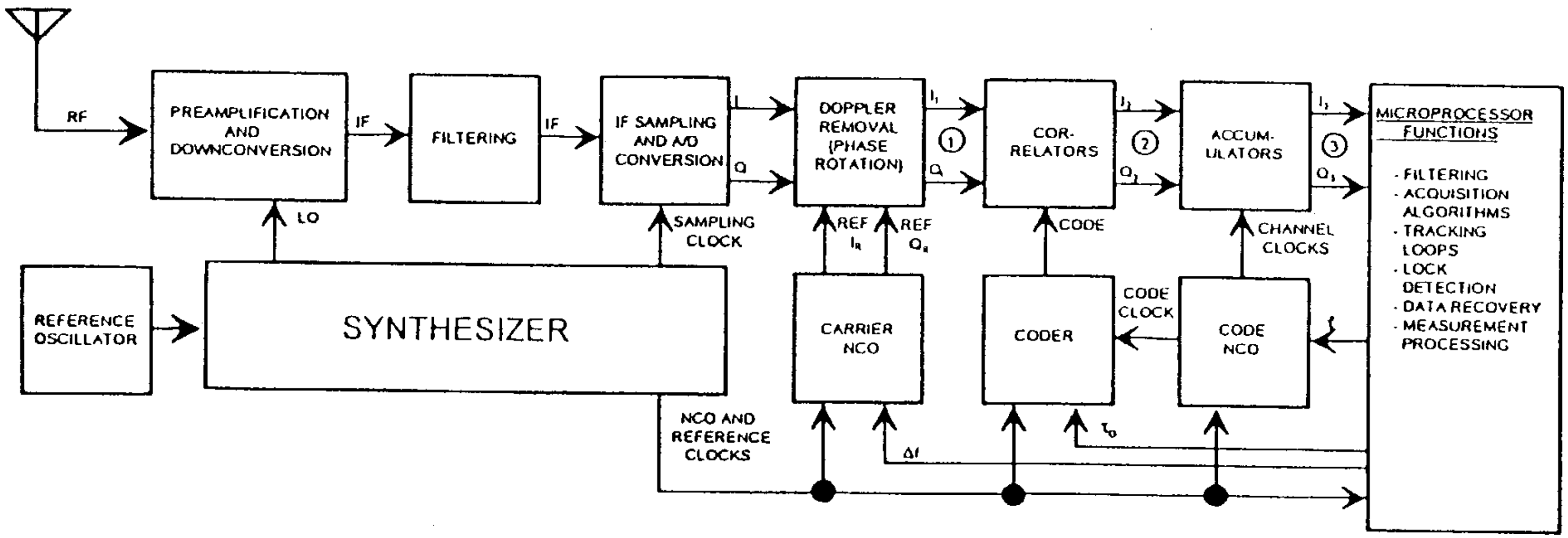
Receiver Functions

- Receives RF signal
- RF (Radio freq.) / IF (Intermediate freq.) down conversion
- Signal Acquisition
  - 2 dimensional search in time and freq. domains
- Signal Tracking
  - DLL for code tracking
  - FLL or PLL for carrier tracking
- Bit & Frame Synchronization
- Generation of PRs, CRs, and DPPLs
- Computation of position, velocity, and time

Example : GPS Receiver

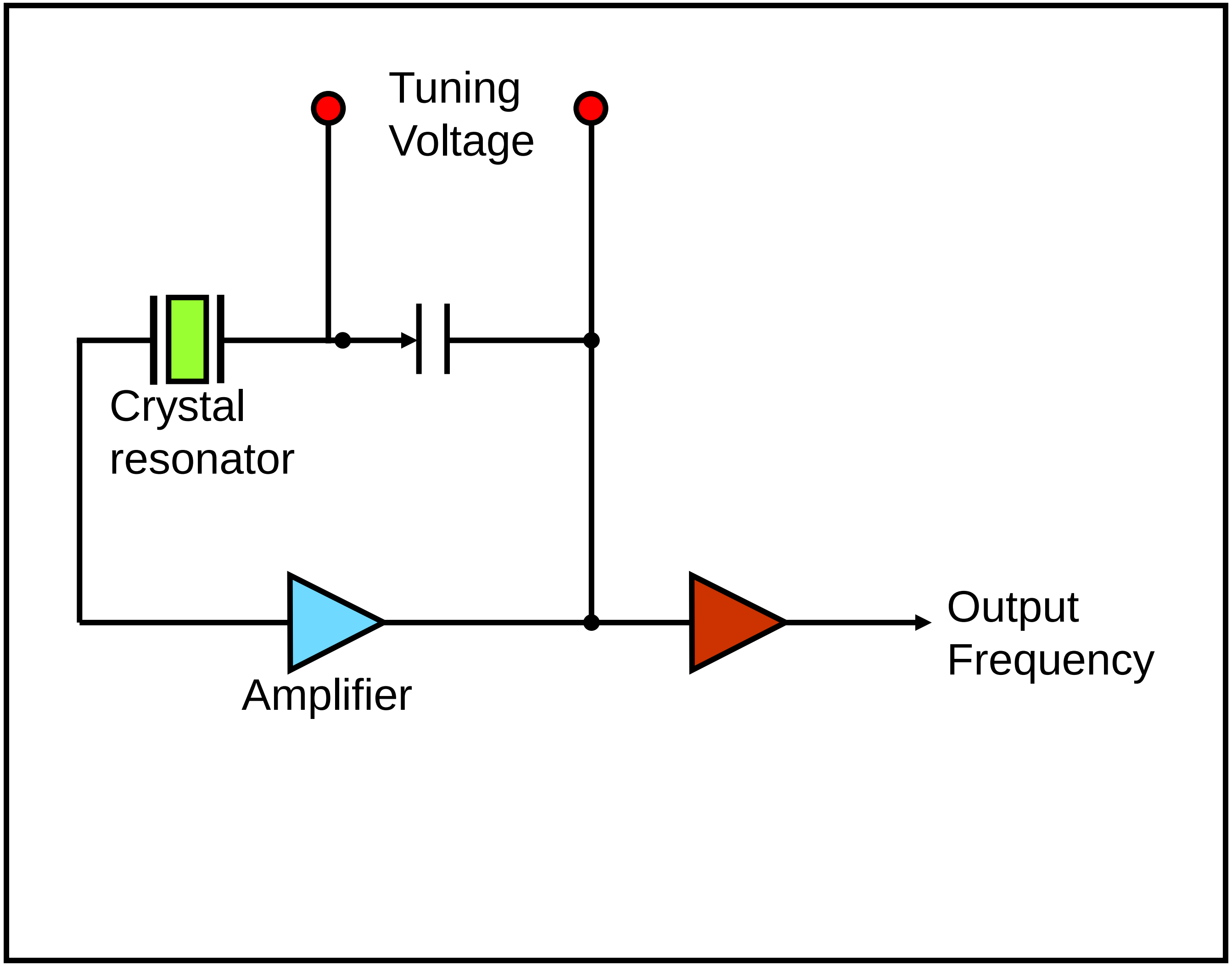
$$A_c d(t)c(t)\sin(\omega_1 t + \phi_c) + A_p d(t)p(t)\cos(\omega_1 t + \phi_{p1}) + A_p d(t)p(t)\cos(\omega_2 t + \phi_{p2})$$



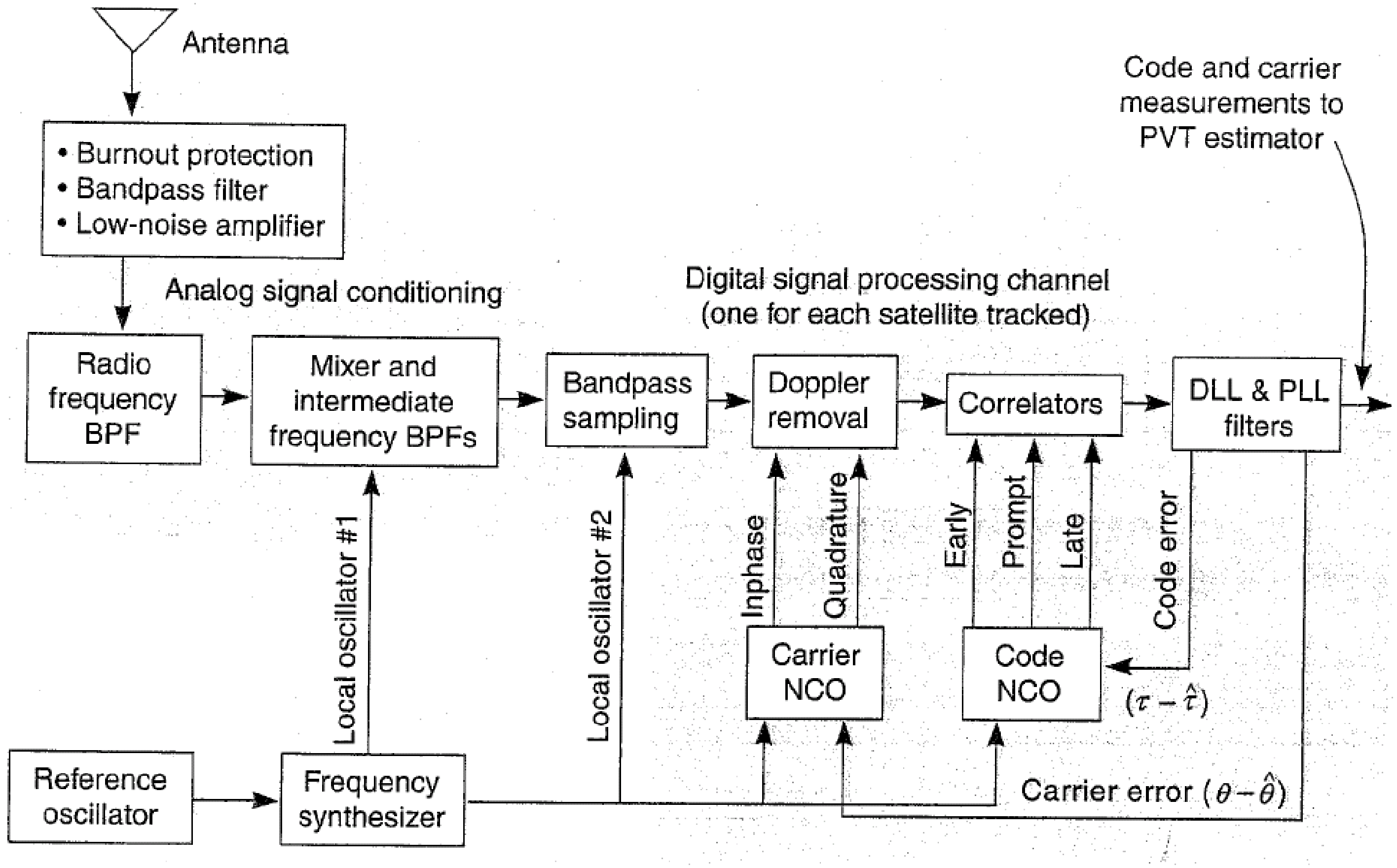


More Detailed Receiver Architecture

Basic Crystal Oscillator



SIGNAL CONDITIONING



Signal processing portions of a receiver

- Super-Heterodyne
  - A "heterodyne" refers to a beat or "difference" frequency produced when two or more external radio frequency carrier waves are fed to a detector. The term was coined by Canadian Engineer Reginald Fessenden in early 1900s.
  - Later, when vacuum triodes became available, the same result could be achieved more conveniently by incorporating a "local oscillator" in the receiver.
  - In December, 1919, Major E. H. Armstrong gave publicity to an indirect method of obtaining short-wave amplification, called the super-heterodyne.

- Frequency down conversion (heterodyning)

$$(A\cos\alpha)(B\cos\beta) = \frac{AB}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

where

$A\cos\alpha = \sqrt{2C}\,x(t-\tau)D(t-\tau)\cos\left[2\pi(f_L + f_D)t+\theta\right]$: received signal

$B\cos\beta = \sqrt{2}\cos\left[2\pi(f_L - f_{IF})t+\theta_{IF}\right]$: receiver-internally-generated signal

τ : elapesed time for the propagation from transmitter to receiver

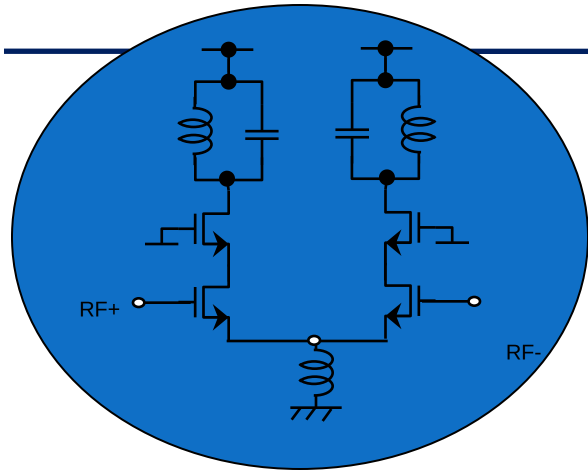
f_L : carrier frequency

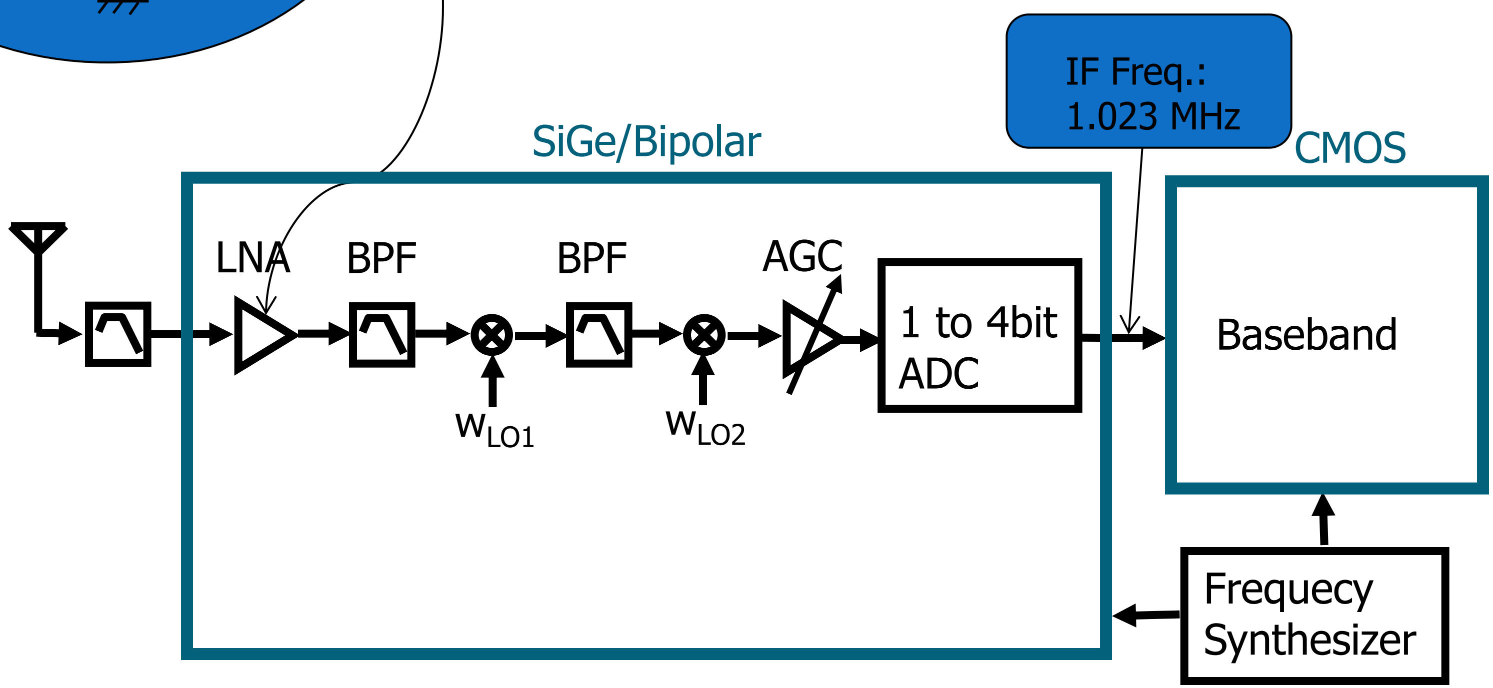
f_D : doppler frequency

f_{IF} : intermediate frequency ($f_{IF} << f_L$)

θ, θ_{IF} : phase offsets

Block Diagram of RF/IF Down Conversion





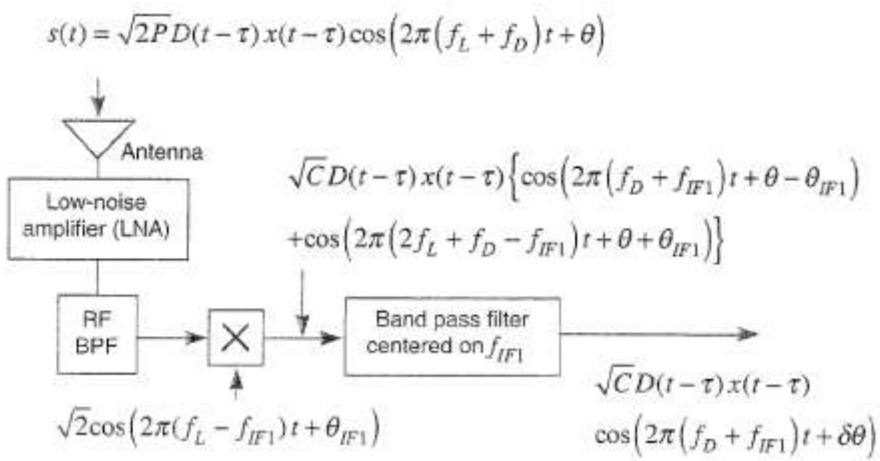
- Mixing
  - Combination of muliplying and filtering
  - Generates a signal modulated by the intermediate freqeuncy

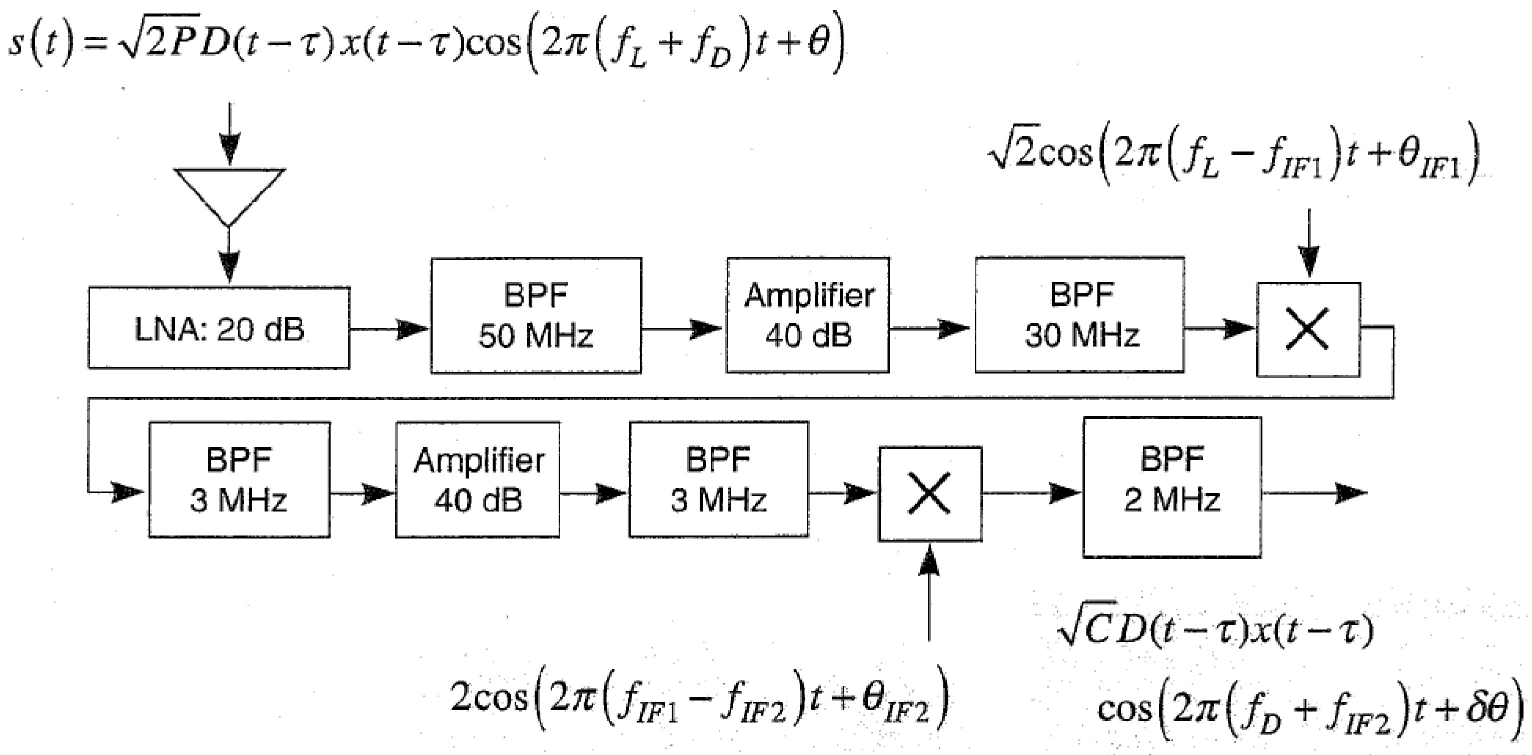
$$\frac{AB}{2}\cos(\alpha+\beta)=\sqrt{C}x(t-\tau)D(t-\tau)\cos\left[2\pi(2f_L - f_{IF} + f_D)t+\theta+\theta_{IF}\right]$$

(removed by a low-pass filter)

$$\frac{AB}{2}\cos(\alpha-\beta)=\sqrt{C}x(t-\tau)D(t-\tau)\cos\left[2\pi(f_{IF} + f_D)t+\theta-\theta_{IF}\right]$$

(down-coverted signal)





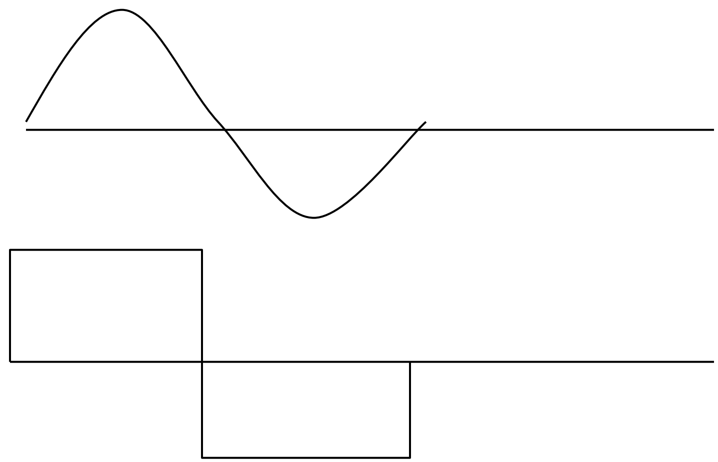
Multiple mixers and intermediate frequency stages are often employed

□ Sampling

- Many receivers use a simple AD converter and these have four levels of quantization
- Such receivers do not suffer much SNR loss relative to a receiver without quantization but they require automatic gain control (AGC)
- The AGC ensures that the signal amplitude is spread among the quatization boundaries of the A/D
- If the sample rate is high enough, then the samples can be used to reconstruct the original signal

–A/D Conversion: One-bit sampling

Signal

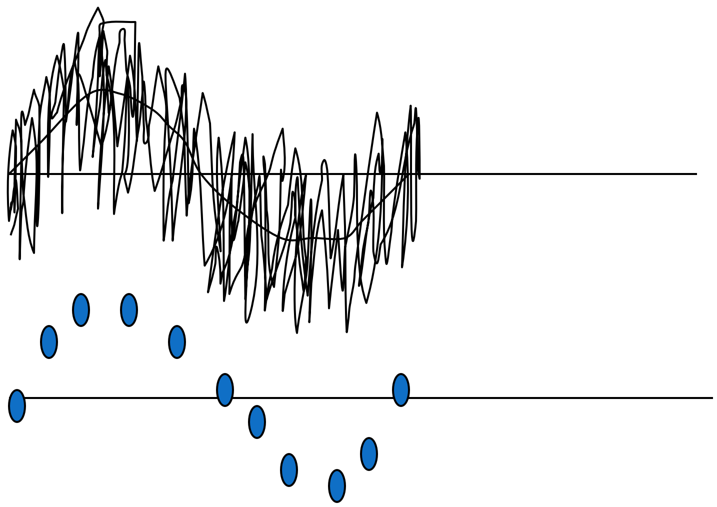


1-bit quantized signal

* Misconception: *One-bit sampling does not provide information on signal amplitude.*

Signal with noise

1-bit quantized signal
[average of many samples]



* In Reality: *With help of noise, ensemble average of one-bit samples gives signal amplitude.*

Qaudrature Signals

- Inphase and Quadrature Processing and Doppler Removal
  - With respect to the input signal

$$\sqrt{C}D(t-\tau)x(t-\tau)\cos\left[2\pi(f_{IF}+f_D)t+\delta\theta\right]$$

  - The receiver internally generates two reference signals

$$\sqrt{2}\cos\left[2\pi(f_{IF}+f_D)t+\theta\right] : \text{inphase reference}$$

$$\sqrt{2}\sin\left[2\pi(f_{IF}+f_D)t+\theta\right] : \text{quadrature reference}$$

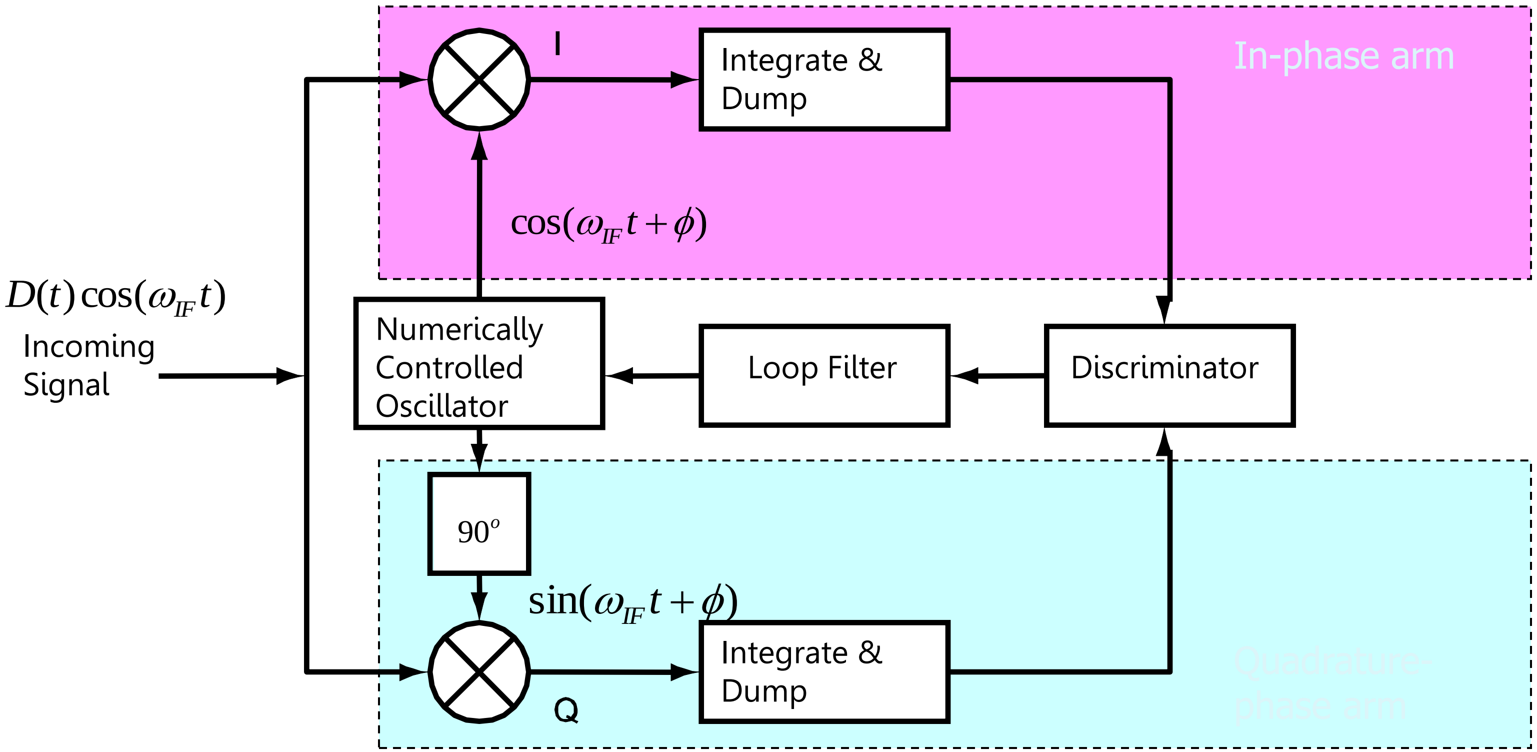
  - After low-pass filtering, the outputs from the inphase and quadrature channels are (carrier wipeoff)

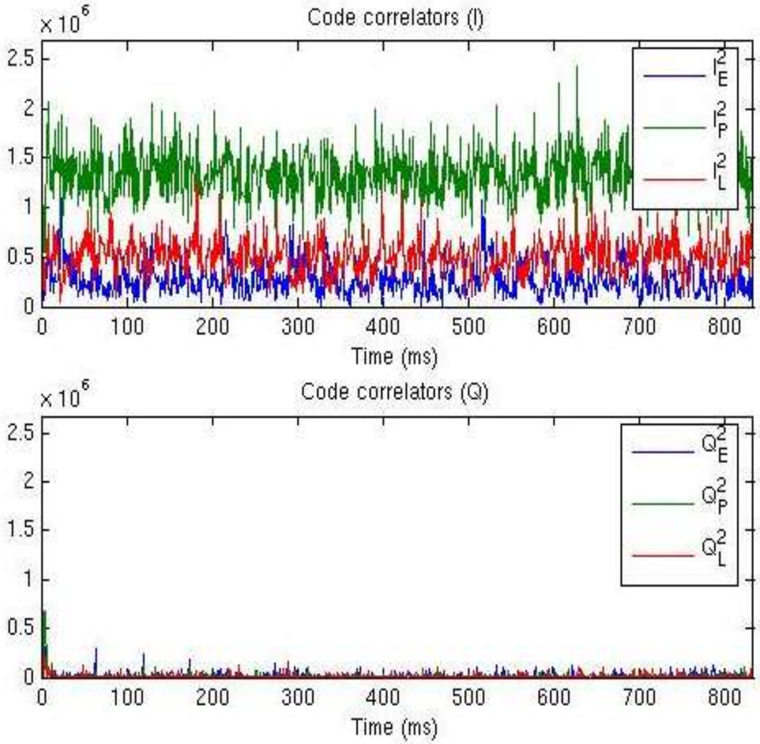
$$\sqrt{C}D(t-\tau)x(t-\tau)\cos(2\pi\Delta f_D t+\Delta\theta) : \text{inphase}$$

$$\sqrt{C}D(t-\tau)x(t-\tau)\sin(2\pi\Delta f_D t+\Delta\theta) : \text{quadrature}$$

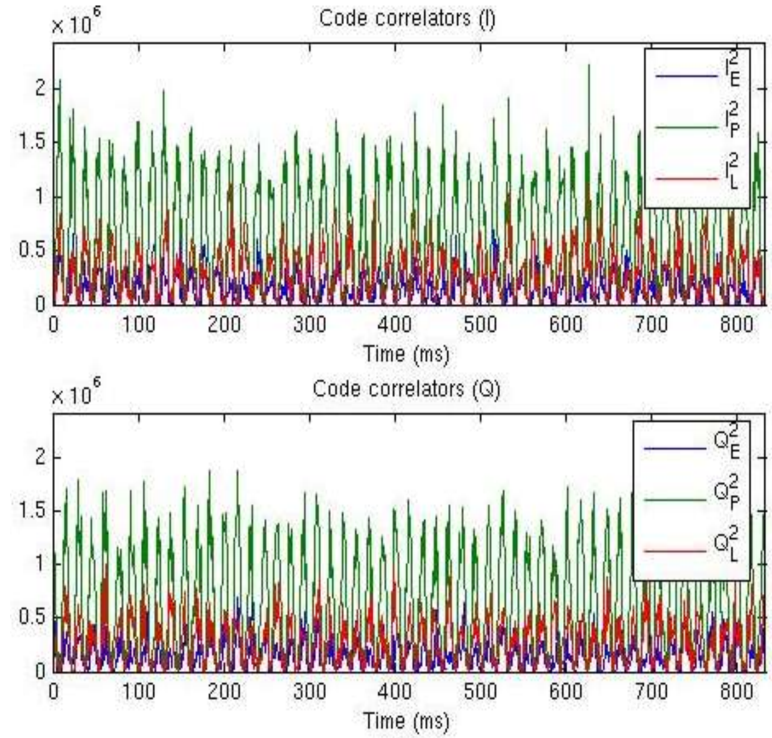
$$\text{where} \quad \Delta f_D = f_D - f_D$$

$$\Delta\theta = \delta\theta - \theta$$


In-phase arm
I
Integrate &
Dump
$\cos(\omega_{IF} t + \phi)$
$D(t)\cos(\omega_{IF} t)$
Incoming
Signal
Numerically
Controlled
Oscillator
Loop Filter
Discriminator
90^o
$\sin(\omega_{IF} t + \phi)$
Integrate &
Dump
Q
Quadrature-
phase arm



Locked State



Unlocked State

Correlator Outputs at (Un)Locked States