ETRI 원내 전문 교육

# GPS/관성센서 통합에 의한 측위 및 응용

## LEC2 GPS FUNDAMENTALS

2005/7/14

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# Overview

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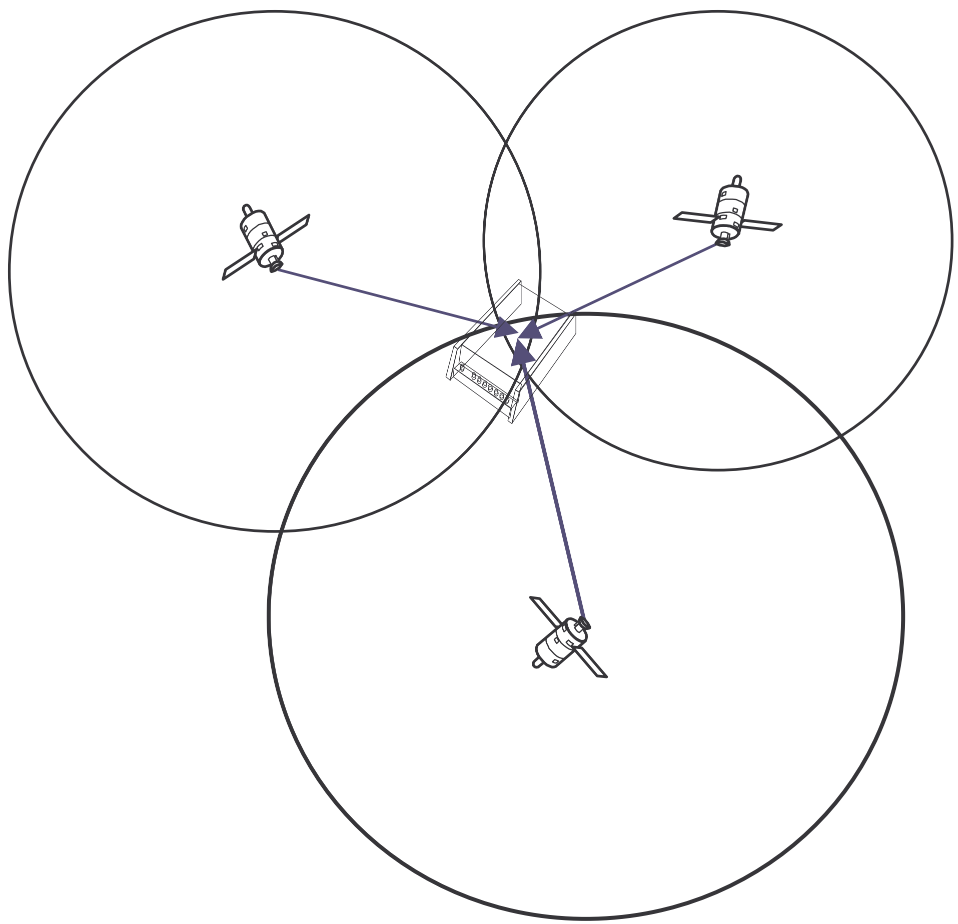
# GPS란?

- 전파항법을 위하여 미국에서 개발한 시스템
- 다수개의 위성과 지상국으로 구성됨
- 위성은 측위 및 항법을 위한 신호 송출
- 지상국들은 위성의 상태 관측 및 성능 관리
- 수신기가 최소 4개 이상의 위성 신호를 받을 경우 삼각법 (trilaterlation)의 원리에 의하여 위치 계산 가능

# 원리 : 삼변법(trilateration)

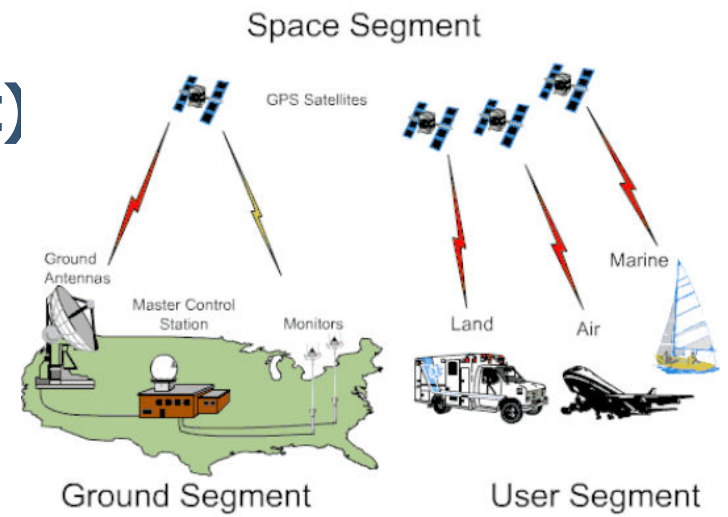


# 연혁

- 1957: Sputnik
- 1960's :
  - VHF Ominidirectional Radios (VOR)
  - LOng-range RAdio Navigation (LORAN)
  - OMEGA
  - Transit (1st satellite-based navigation system)
- Early 1970's :
  - Timation (tested atomic clock)
  - 621B (tested ranging method by PseudoRadom Noise)
- 1973 : GPS program begins

# GPS 구성 요소의 종류

- 우주 부분 (space segment)
  - 위성의 개발, 제조, 발사
- 지상 부분 (ground segment)
  - 각 위성의 성능 및 상태 모니터
  - 각 위성의 위치 및 시간 정보 보정치 계산
- 사용자 부분 (user segment)
  - 위성신호 수신
  - 수신기의 위치 및 시간 계산





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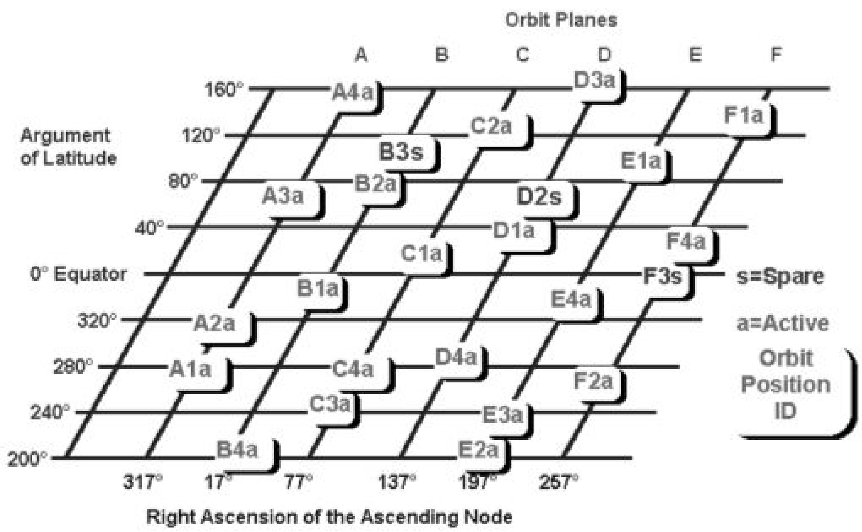
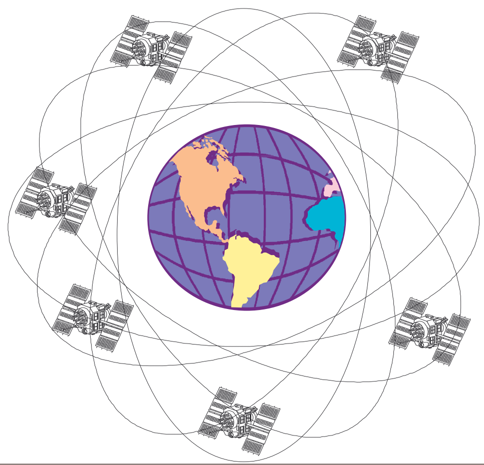
# GPS 구성 요소: 우주 부분

- 위성 궤도
  - 24 + 4 (예비)
  - 12시간 주기
  - 6개의 궤도평면
  - 55도의 경사각




- 가시성
  - 지구 전역에서 6~ 8 개의 위성 관측 가능



- 신호 특성
  - 2주파수 (L1, L2)
  - 확산대역 (spread spectrum)



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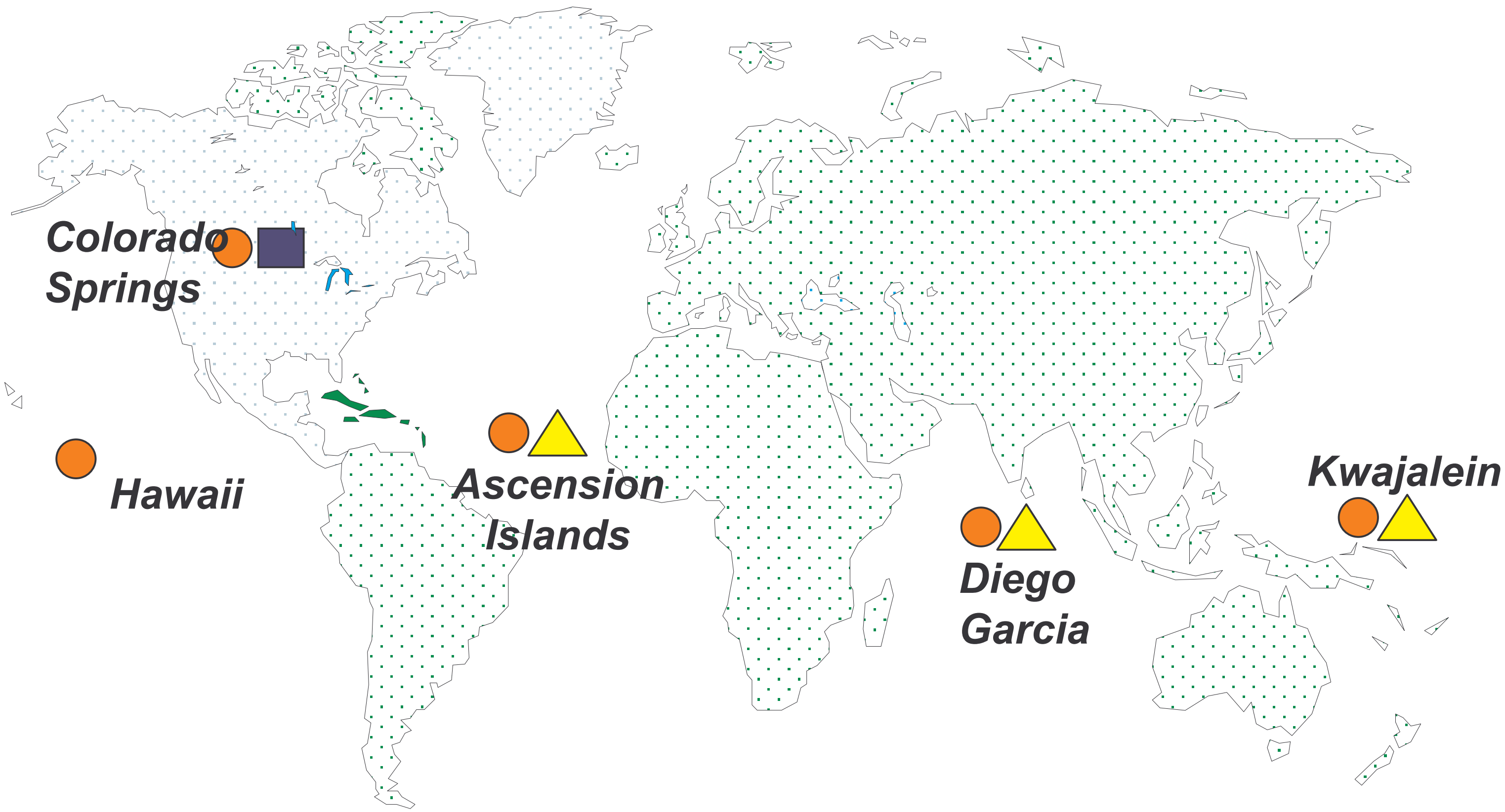
# GPS 구성 요소: 지상 부분




■ **Master Control Station (1)** : observe ephemeris and clock

● **Monitor Station (5)** : correct orbit and clock errors, create new navigation messages

▲ **Ground Antenna (3)** : upload corrections and messages



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# Pseudo Random Noise

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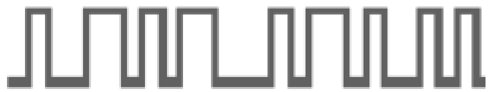
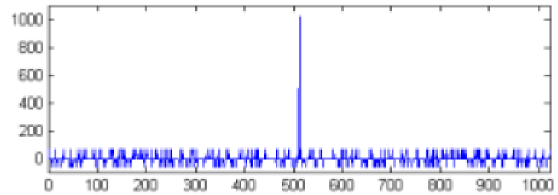
# *PseudoRandom Noise (PRN) ?*

- 인위적으로 설계된 직교성을 가진 순열(1과 0으로 구성)
  - PRN#j(t) = [ 1 0 0 1 0 1 1 1 … $]^T$



- GPS 각 위성에는 고유한 PRN 번호가 부여됨

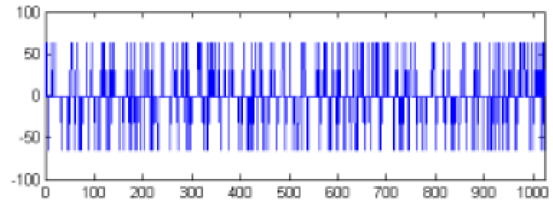
- i-번째와 j-번째 위성을 고려하면




  - autocorrelation

    PRN#i(t)$^T$ PRN#i(t-a) = non-zero constant if a = 0

    almost zero else

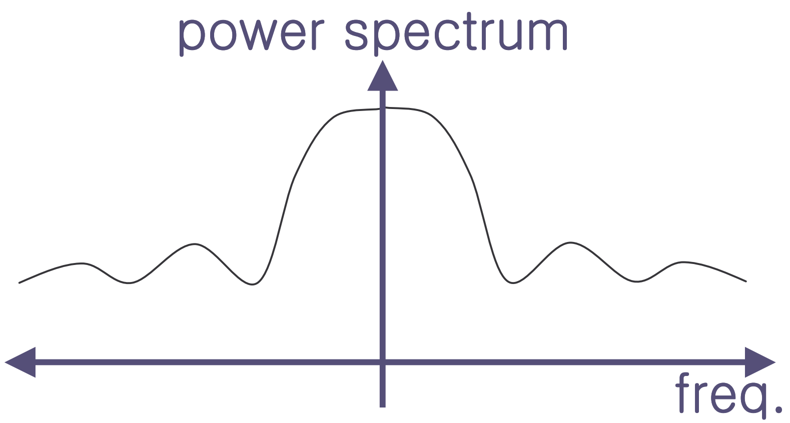



  - crosscorrelation

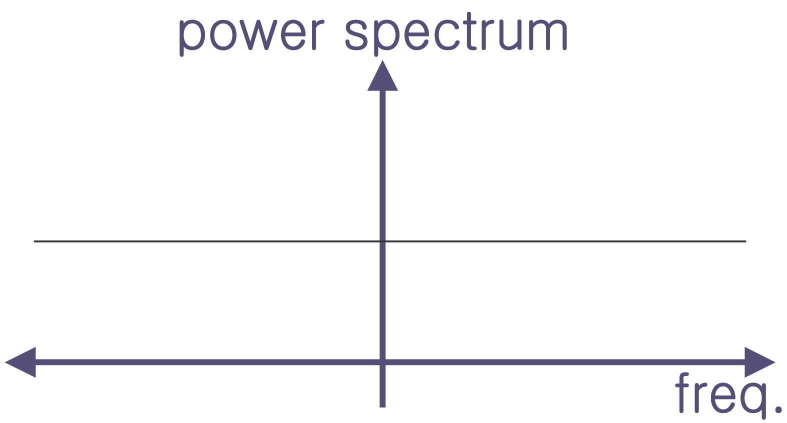
    PRN#i(t)$^T$ PRN#j(t-a) = almost zero always

# 주파수 영역에서 PRN과 백색잡음의 비교

- PRN



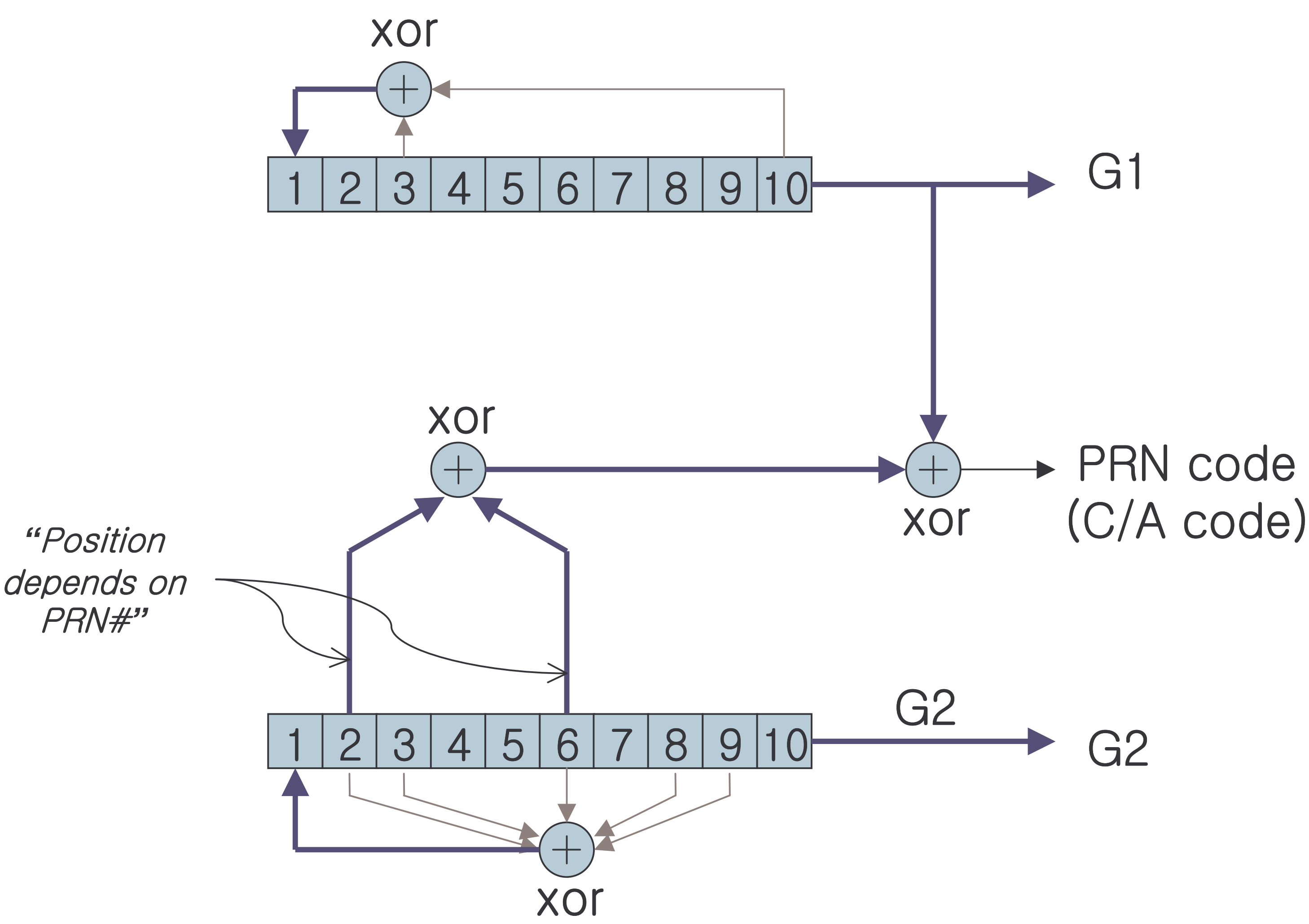

- 백색잡음



# GPS PRN 코드의 종류

- C/A 코드
  - Coarse Acquisition code
  - 민간용
  - 1 ms 주기
- P 코드
  - Precise code
  - 군용
  - 267 일 주기

# PRN 생성 구조



xor
1 2 3 4 5 6 7 8 9 10
G1
xor
xor
PRN code
(C/A code)
"Position
depends on
PRN#"
G2
1 2 3 4 5 6 7 8 9 10
G2
xor

# Signal Generation and Transmission

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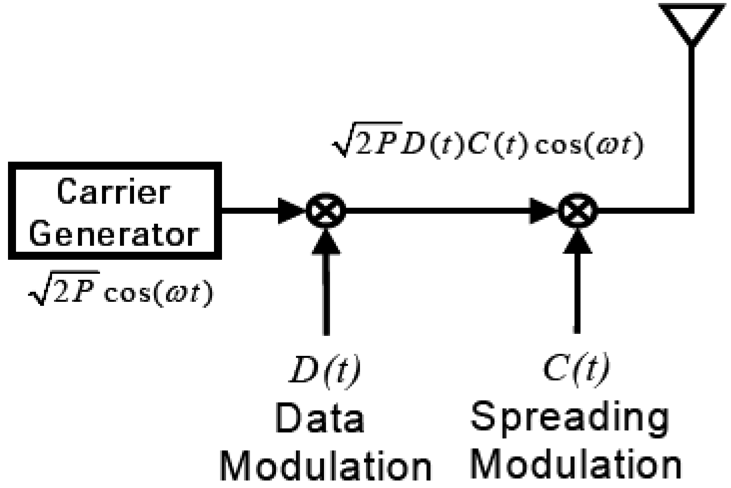
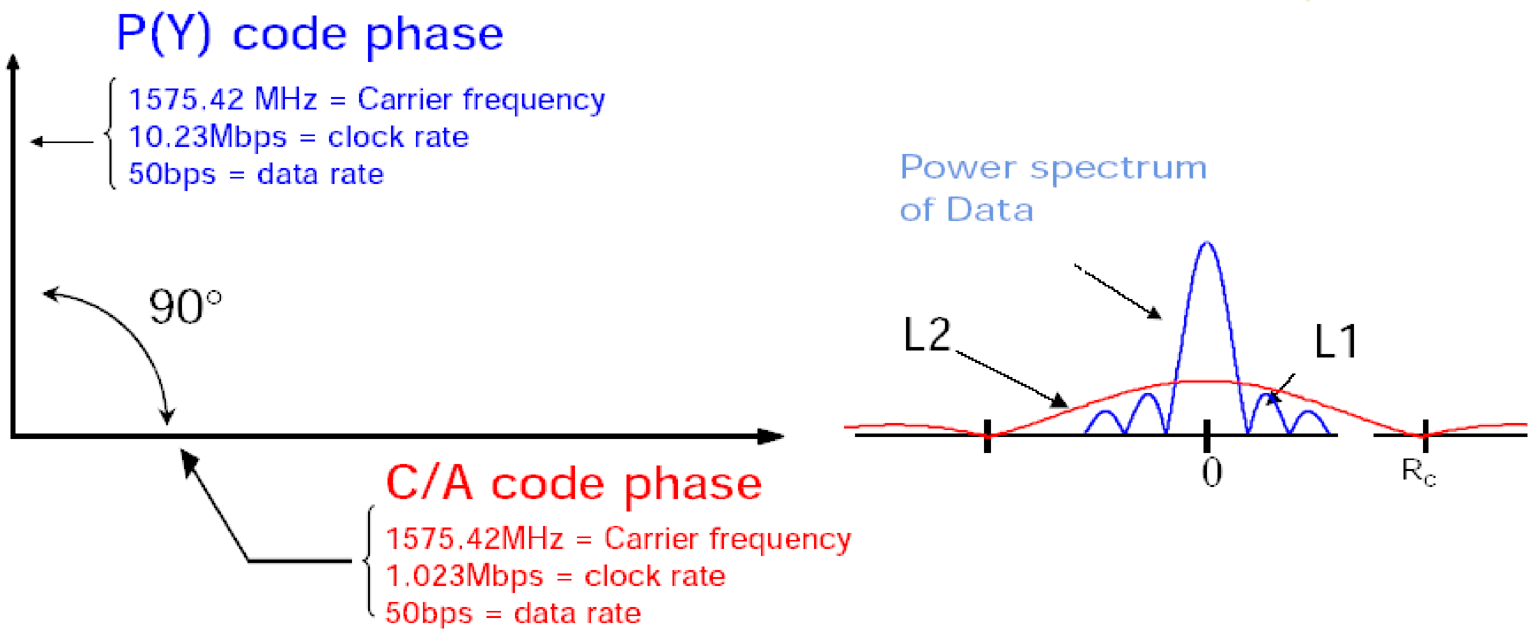
# GPS 신호 개관

- **Fundamental Frequency**
  - $f_0$ = 10.23 MHz
- **Carrier Frequencies**
  - $f_{L1}$ = $154f_0$ = 1575.42 MHz
  - $f_{L2}$ = $120f_0$ = 1227.60 MHz
- **C/A Code on L1 Carrier**
  - $f_{C/A}$ = $f_0$ /10, 1 ms period
- **P Code on L2 Carrier**
  - $f_P$ = $f_0$ , 267 days period
- **Navigation Message**
  - 1500 bit sequence, 50 bps
- **BPSK (Binary Phase Shift Keyed) Modulation**



# C/A Code on L1 VS P Code on L2



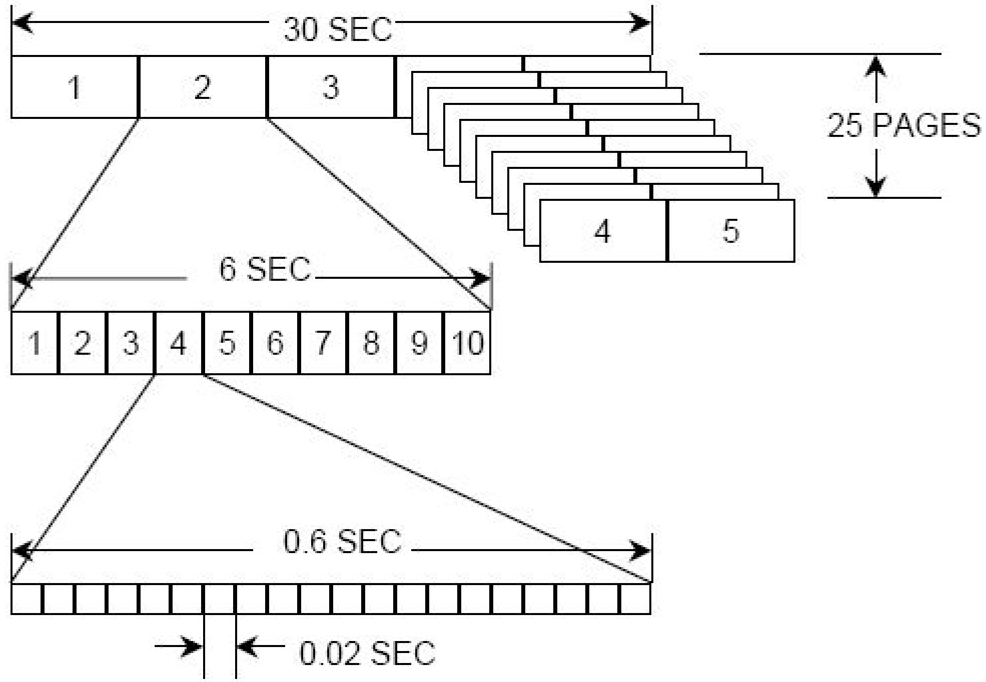
# *Navigation Message*

BASIC MESSAGE UNIT IS ONE FRAME ( 1500 BITS LONG )

1 FRAME = 5 SUBFRAMES

1 SUBFRAME = 10 WORDS

1 WORD = 30 BITS



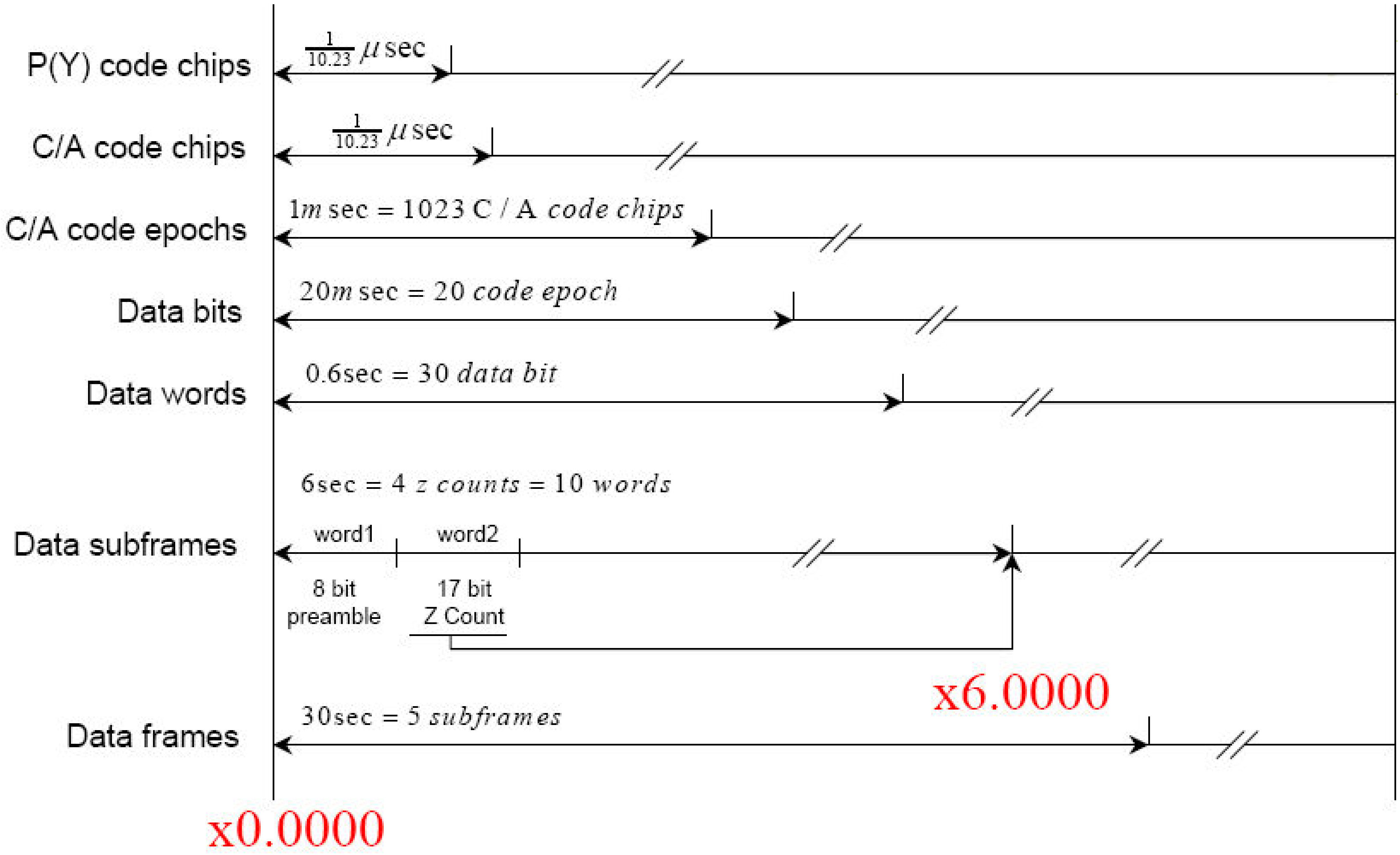

ONE **MASTER FRAME** INCLUDES
ALL 25 PAGES OF SUBFRAMES 4 & 5 = 37,500 BITS TAKING 12.5 MINUTES

Ten, 30-bit words, 6-s subframe

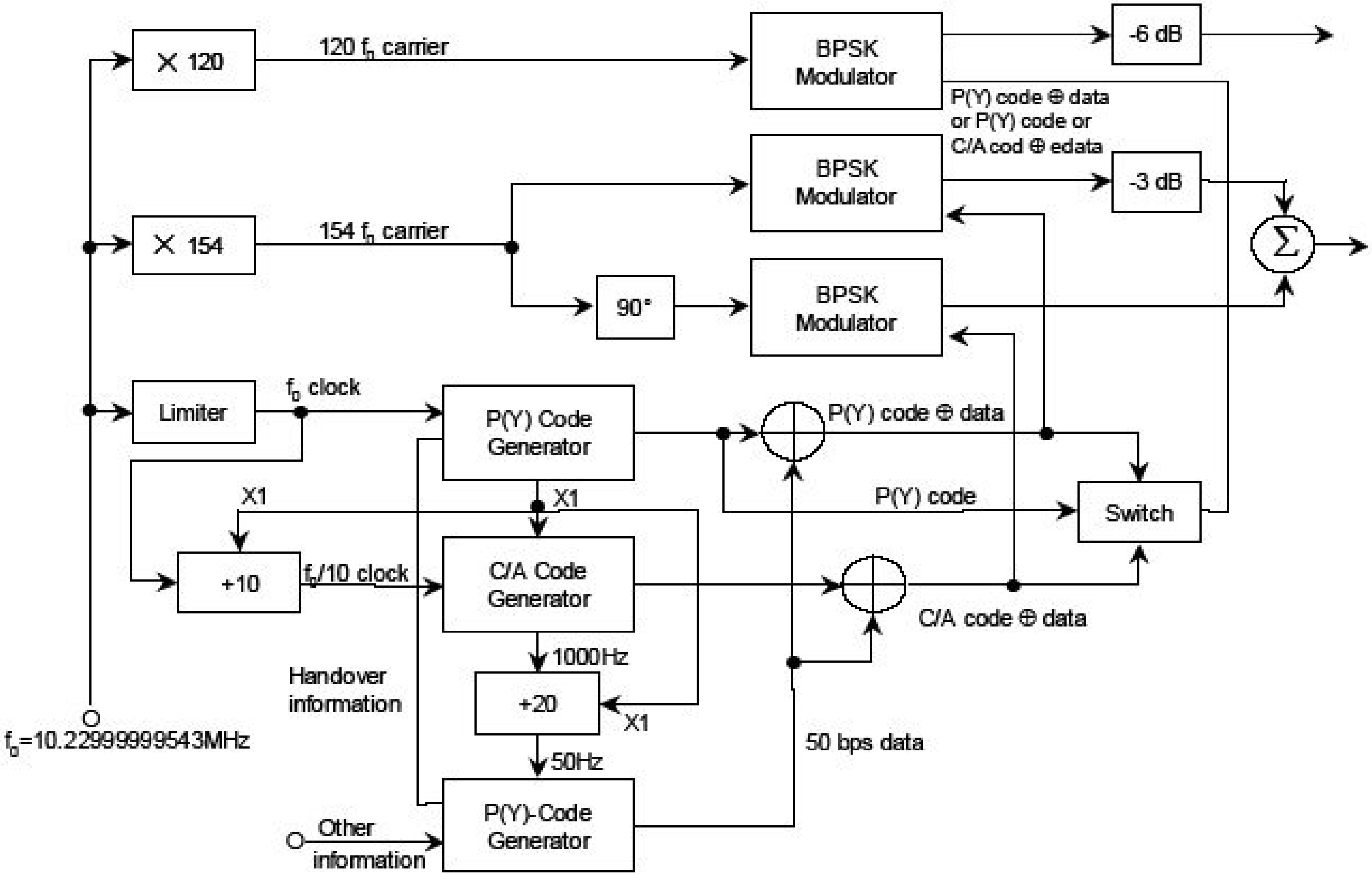
| Subframe No. | | | |
|---|---|---|---|
| 1 | TLM | HOW | Block 1 - Clock correction + satellite quality |
| 2 | TLM | HOW | Block 2 -Ephemeris |
| 3 | TLM | HOW | Block 1 - Ephemeris continued |
| 4 | TLM | HOW | Block 1 - Almanac+ionosphere+UTC correction |
| 5 | TLM | HOW | Block 1 - Almanac-(25frames for complete almanac) |

1-frame
30s
1500 bits

# Timing Relationship



# 위성에 의한 GPS 신호 생성 구조



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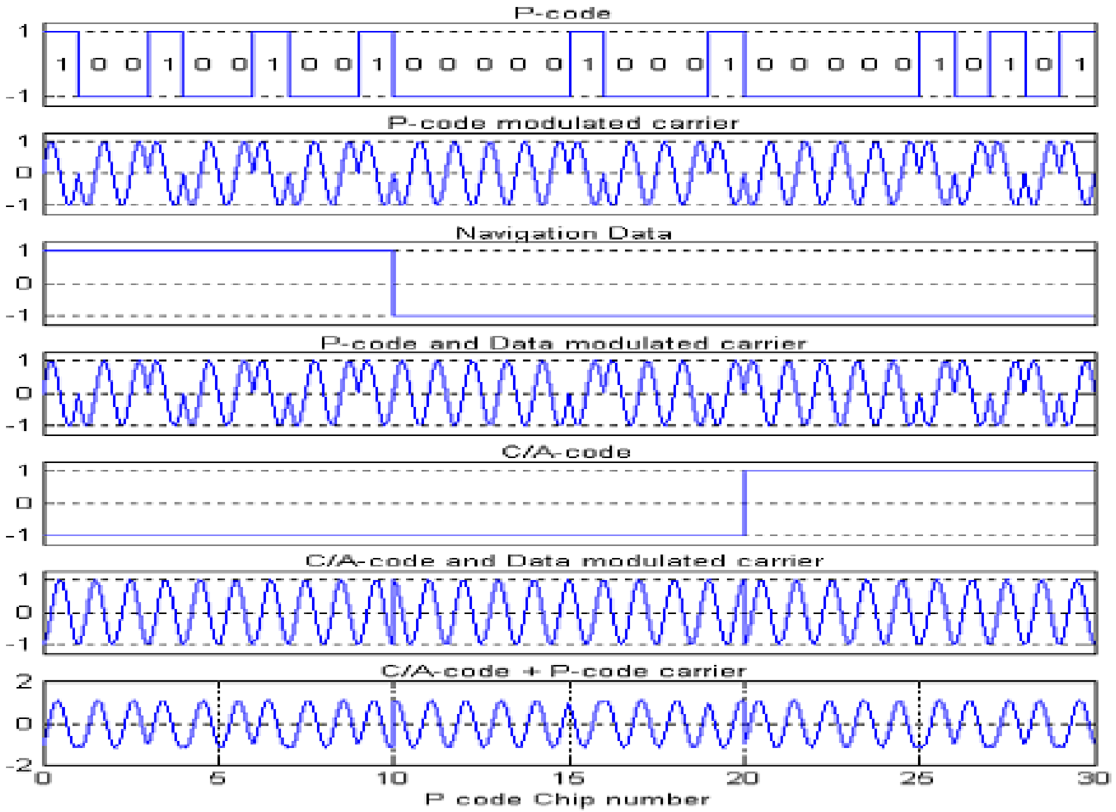
# 변조된 GPS 신호의 파형

- Navigation Data, C/A 코드, 그리고 P 코드는 BPSK 변조된다.
- 논리 0 은 위상변화 0 도에 해당. 원래의 L1(L2) carrier 파형 유지
- 논리 1 은 위상변화 180 도에 해당. 원래의 L1(L2) carrier 파형 반전

$$A_p d(t) p(t) \cos(\omega_1 t + \phi_{p1})$$

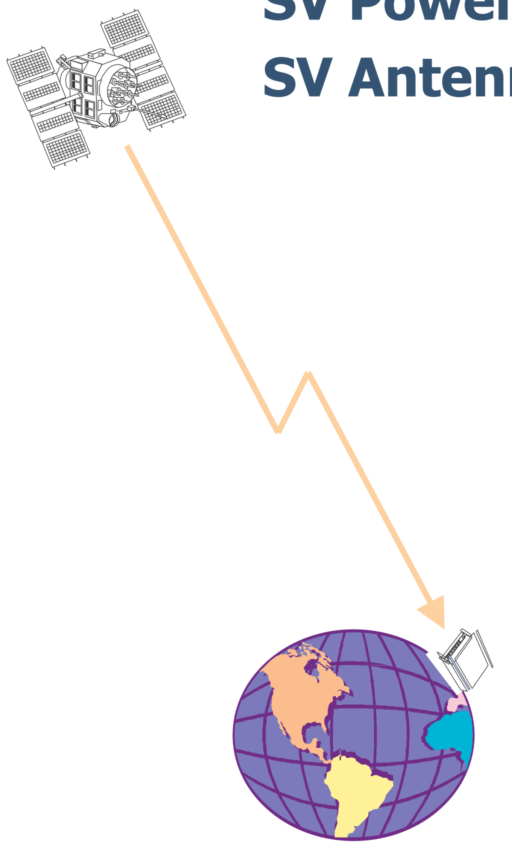
$$A_c d(t) c(t) \sin(\omega_1 t + \phi_c)$$



# *Power Level*

SV Power : $P_T = 27\ W$

SV Antenna Gain : $G_T = 10 \sim 16$



**Received Power**

$= ( P_T\, G_T\, G_A ) / (4 \pi R^2 )$

$= 10^{-16W}$

$= - 130\ dBm$

# Signal Reception

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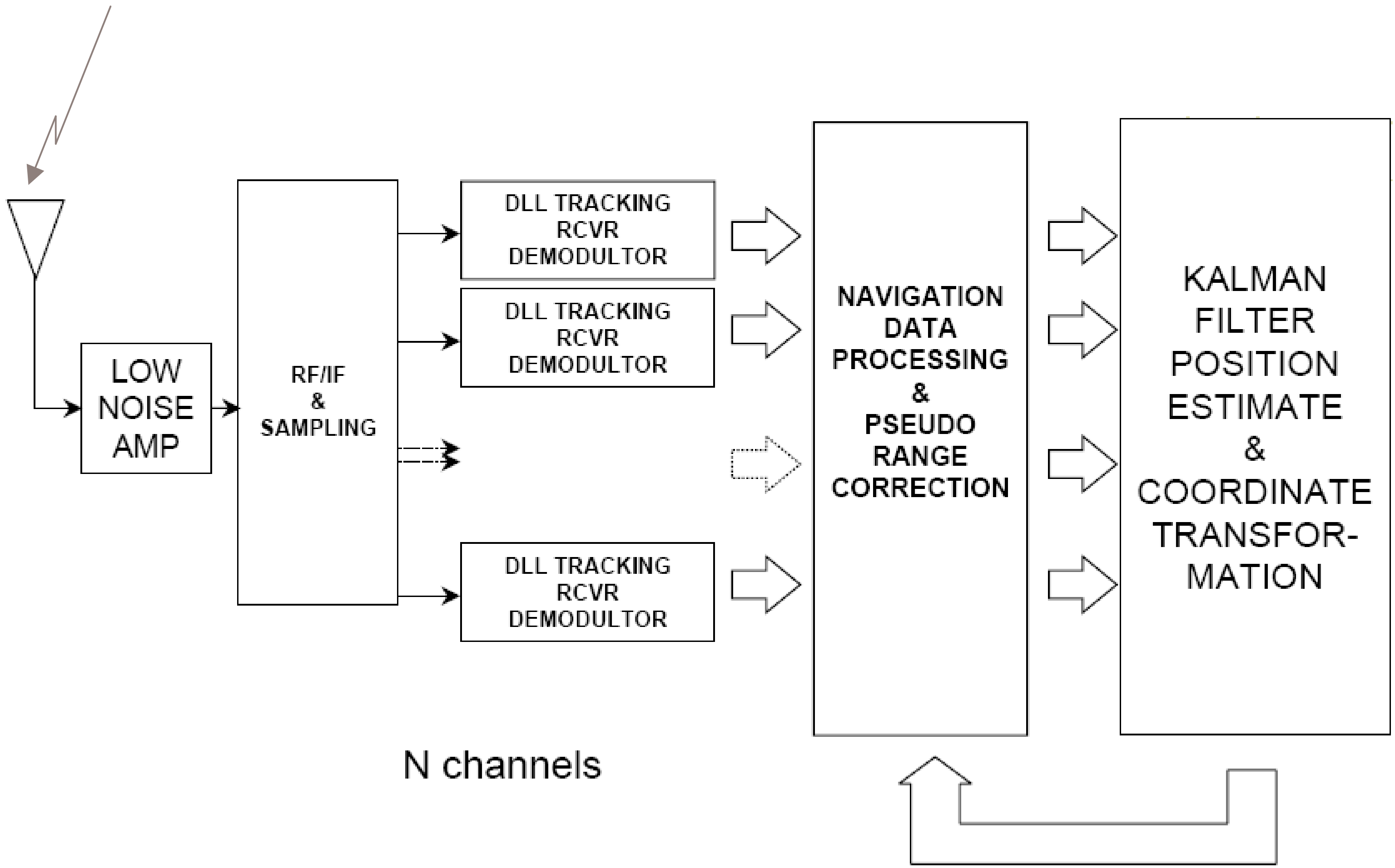
# GPS 수신기의 기능

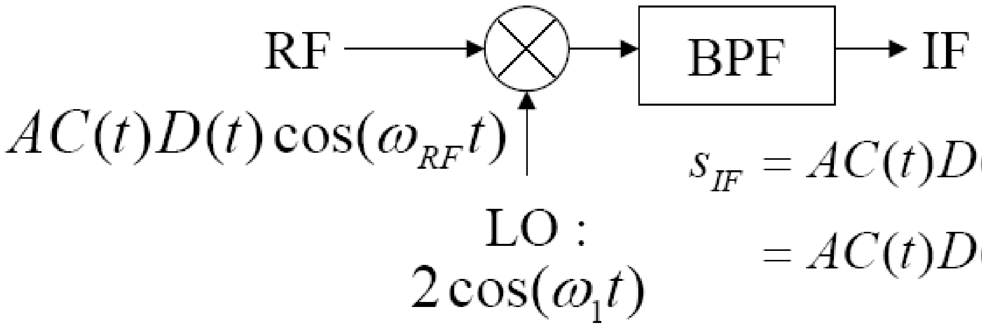
- 위성 신호 수신
- RF (radio freq.) / IF (intermediate freq.) down conversion
- Signal Acquisition
  - 2 dimensional search in time and freq. domains
- Signal Tracking
  - DLL for code tracking
  - FLL or PLL for carrier tracking
- Bit & Frame Synchronization
- 의사거리(pseudorange) 생성 (DLL)
- 누적위상 및 도플러 측정치 생성 (FLL or PLL)
- 의사거리, 누적위상, 도플러 등을 이용한 위치 및 속도 계산

# GPS 수신기의 구조

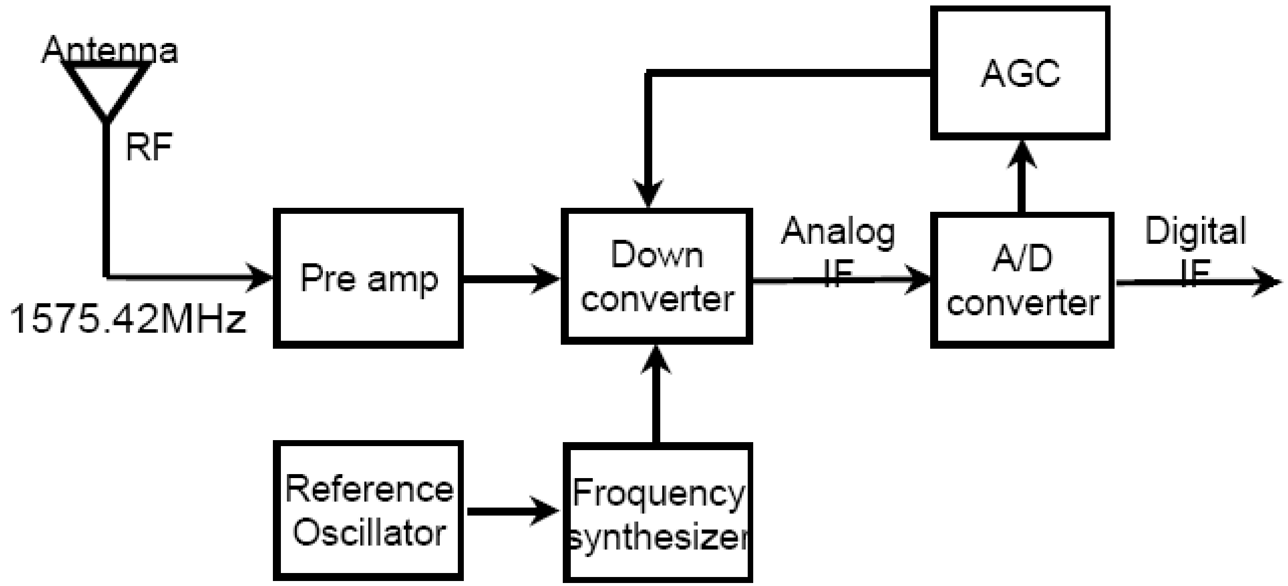
$$A_c d(t)c(t)\sin(\omega_1 t + \phi_c) + A_p d(t)p(t)\cos(\omega_1 t + \phi_{p1}) + A_p d(t)p(t)\cos(\omega_2 t + \phi_{p2})$$



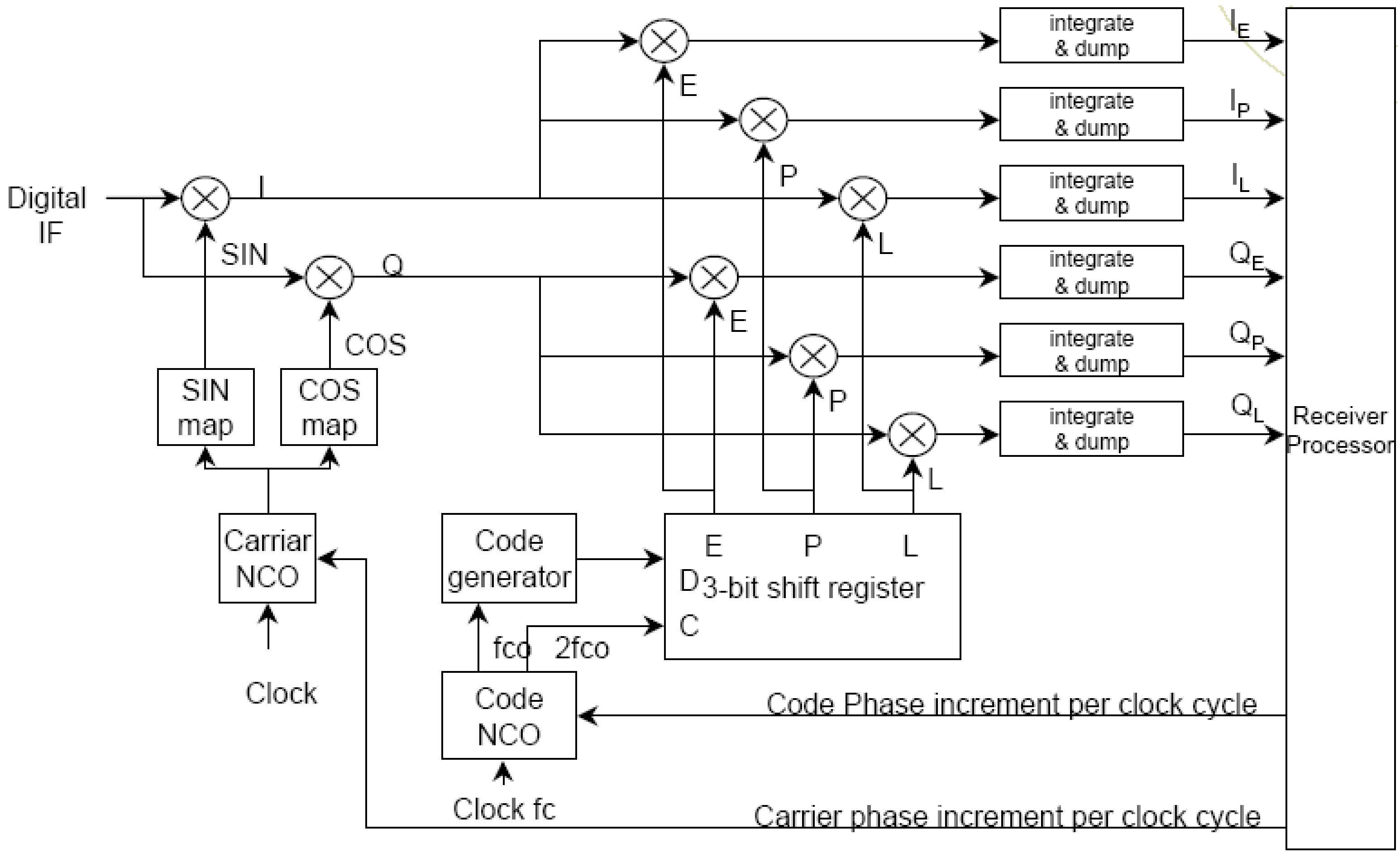
# RF/IF Down Conversion




$$
\begin{aligned}
s_{IF} &= AC(t)D(t)\cos\left[(\omega_0 - \omega_1 + \Delta\omega)t + \phi_0\right] \\
&= AC(t)D(t)\cos\left[(\omega_{IF} + \Delta\omega)t + \phi_0\right]
\end{aligned}
$$



# Correlator and Oscillator



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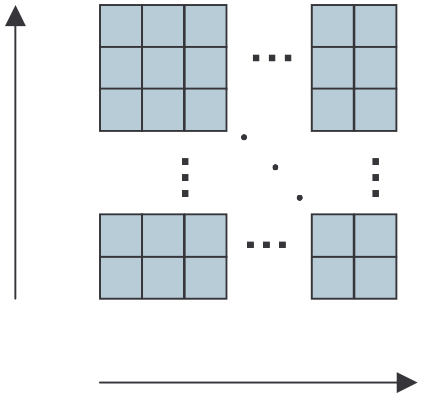
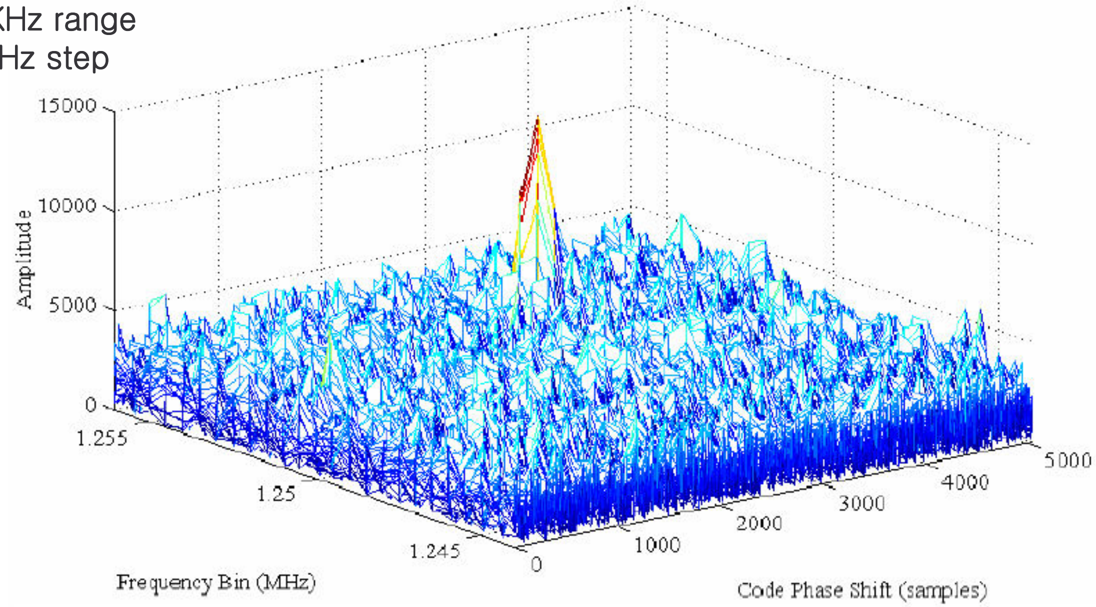
# Signal Acquisition

delay search
- 2046 chip range
- 0.5 chip step



freq. search
- 100 KHz range
- 0.5 KHz step



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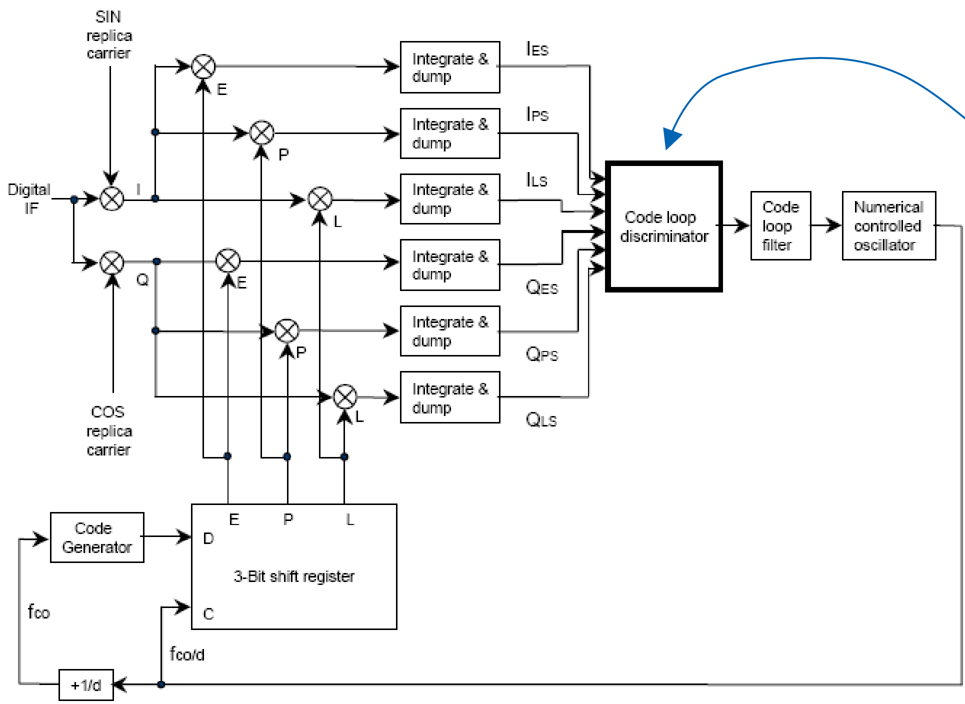
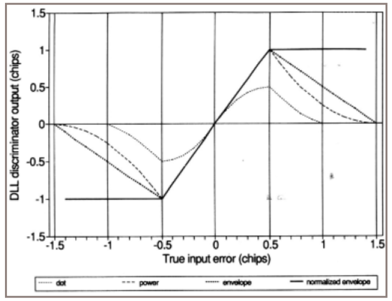
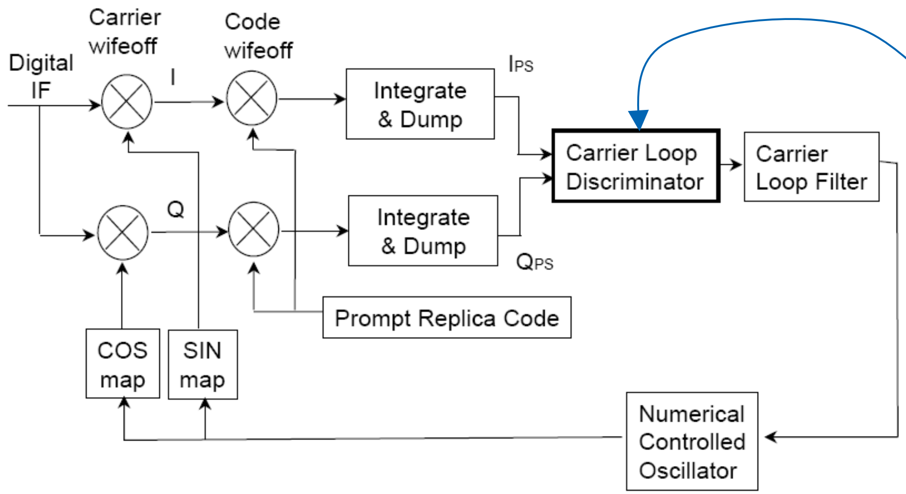
# DLL for Code Tracking



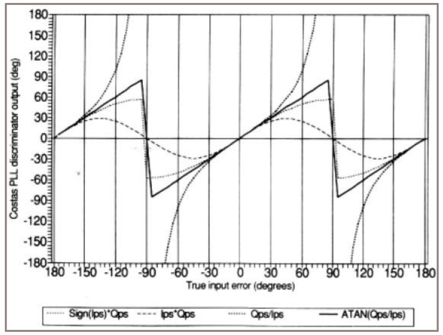

| *Discriminator Algorithm* |
|---|
| $\sum (I_{ES} - I_{LS})I_{PS} + \sum (Q_{ES} - Q_{LS})Q_{PS}$ |
| $\sum (I_{ES}^2 + Q_{ES}^2) - \sum (I_{LS}^2 - Q_{LS}^2)$ |
| $\sum \sqrt{(I_{ES}^2 + Q_{ES}^2)} - \sum \sqrt{(I_{LS}^2 + Q_{LS}^2)}$ |
| $\dfrac{\sum \sqrt{(I_{ES}^2 + Q_{ES}^2)} - \sum \sqrt{(I_{LS}^2 + Q_{LS}^2)}}{\sum \sqrt{(I_{ES}^2 + Q_{ES}^2)} + \sum \sqrt{(I_{LS}^2 + Q_{LS}^2)}}$ |



# PLL or FLL for Carrier Tracking

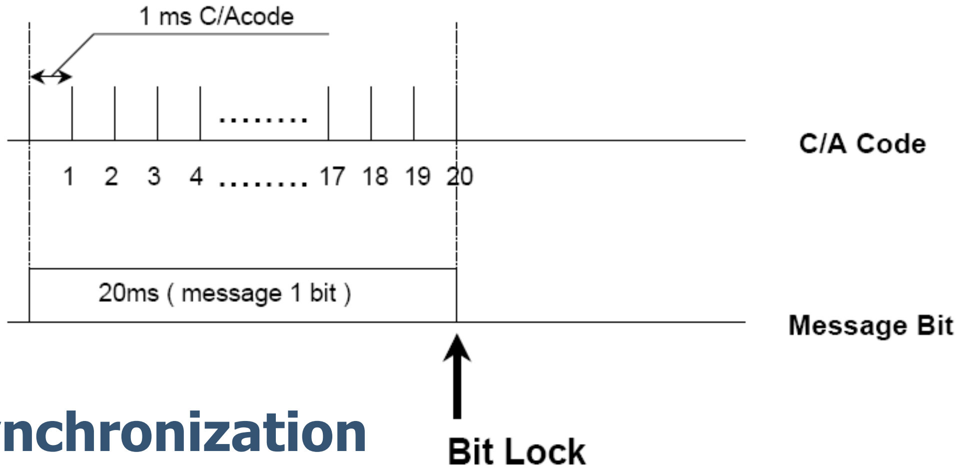



| *Discriminator Algorithm* | *Output Phase Error* |
|---|---|
| Sign $(I_{PS}) \cdot Q_{PS}$ | $\sin\phi$ |
| $I_{PS} \cdot Q_{PS}$ | $\sin 2\phi$ |
| $Q_{PS} / I_{PS}$ | $\tan\phi$ |
| $ATAN(Q_{PS} / I_{PS})$ | $\phi$ |



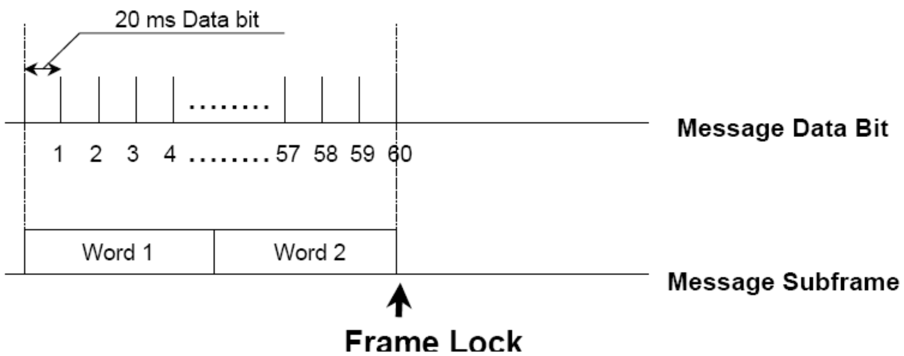
# Bit & Frame Synchronization

- **Bit Synchronization**

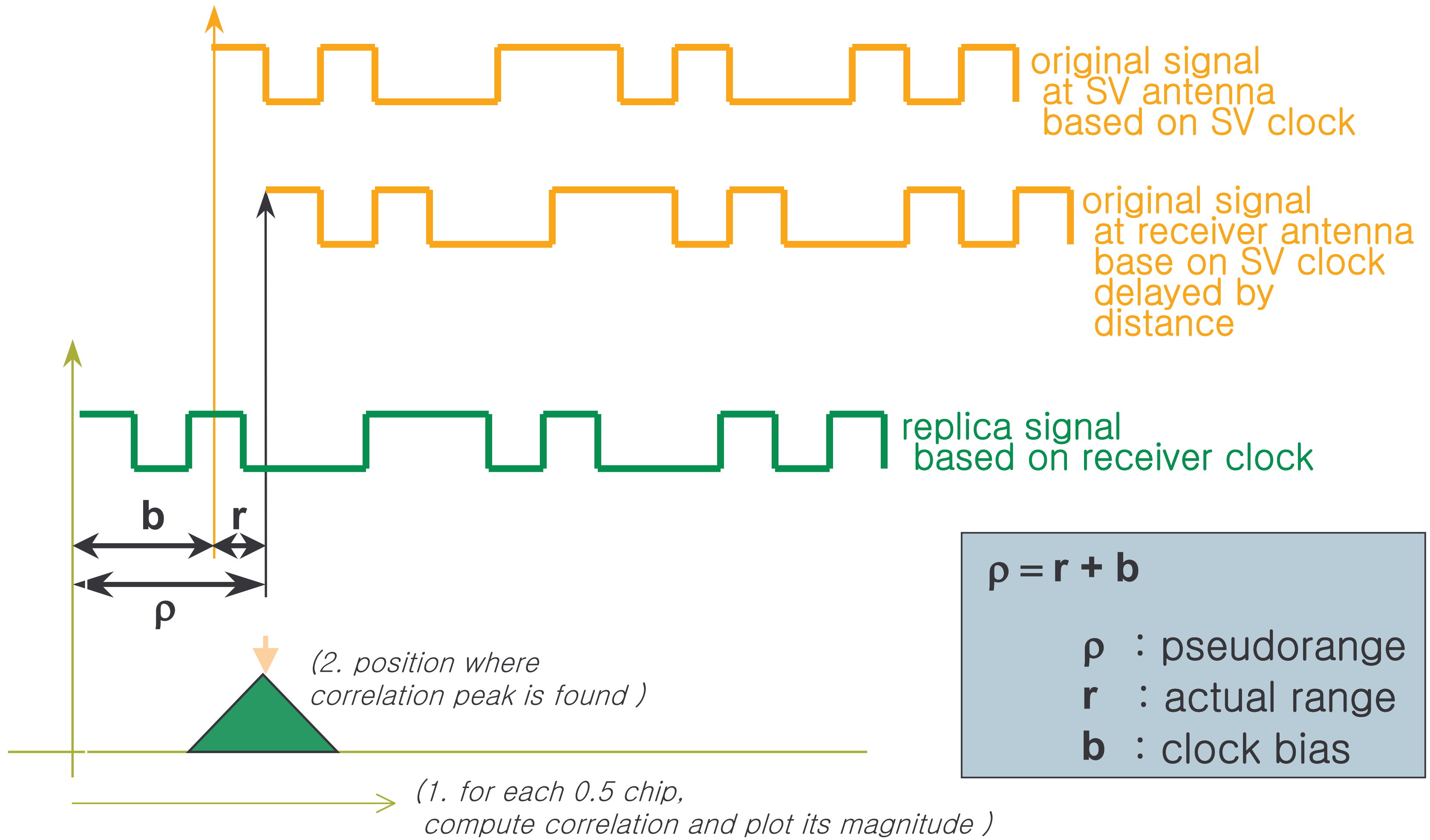



- **Frame Synchronization**

1. TLM preamble (10001011)
2. HOW subframe ID (1 to 5)
3. HOW zero bits (bits 29 and 30)
4. Parity check



# Pseudorange ?




$\rho = r + b$

- $\rho$ : pseudorange
- $r$ : actual range
- $b$ : clock bias

# Random Variables

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# What To Estimate ?

$$X_k := \begin{bmatrix} x_{u,k} \\ \cdots\cdots \\ b_{u,k} \end{bmatrix}$$ state

$$\Delta X_k := X_{k+1} - X_k = \begin{bmatrix} \Delta x_{u,k} \\ \cdots\cdots\cdots \\ \Delta b_{u,k} \end{bmatrix}$$ incremental state

$$\rho_{j,k} = \left\| x_{u,k} - x_{j,k} \right\| + b_{u,k}$$ pseudorange(PR; range+clock bias) w.r.t. the j-th S

$$\Delta \rho_{j,k} = \rho_{j,k+1} - \rho_{j,k}$$ incremental PR w.r.t. the j-th SV

where

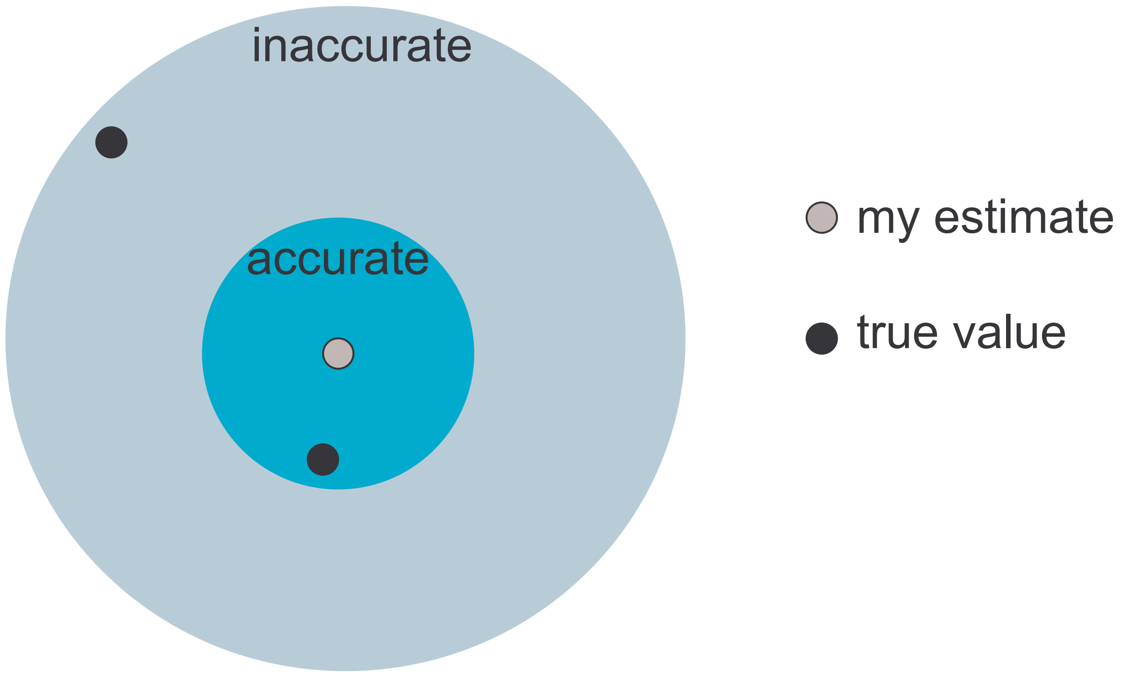
$x_{u,k}$ : ECEF receiver position

$x_{j,k}$ : j-th SV's ECEF position

$b_{u,k}$ : receiver clock bias

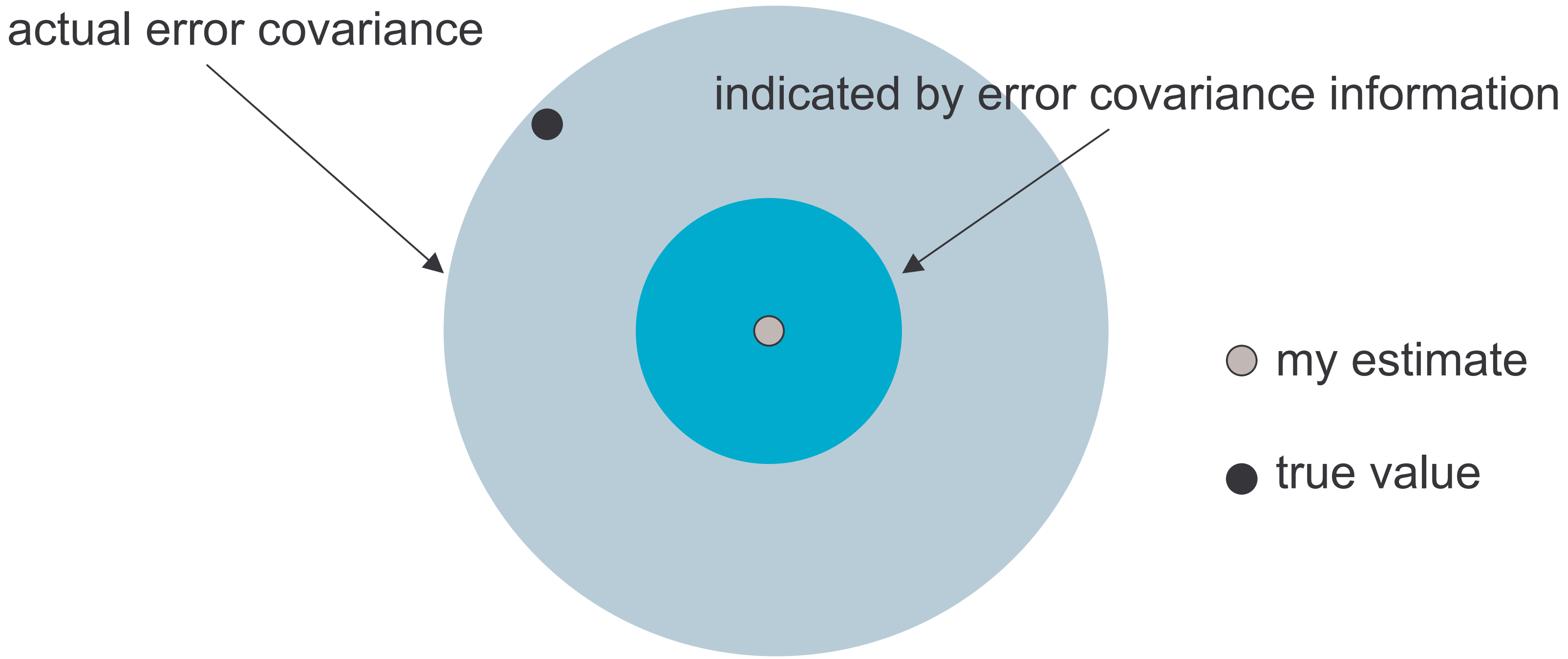
# *Why error covariance is required ?*

To manifest how accurate my estimate is.



# Why consistent error covariance is required ?

“ Exaggeration on accuracy may harm other people “



# Utilization of error covariance

* Accurate estimation (all application)

* Reliable signal quality monitoring (fault detection)

* Initialization of float ambiguity (precise positioning)

* Validation of integer ambiguity (precise positioning)

# Scalar Random Variable

$\{x_i\}_{i=1,2,3,\ldots}$ : samples of discrete white Gaussian random variable $X \sim (m_X, r_X)$

* Expectation (mean)

$$m_X := E[x] = \frac{\sum_{i=1}^{N} x_i}{N}$$

* Variance

$$r_X := E[(X - m_X)(X - m_X)] = \frac{\sum_{i=1}^{N} (x_i - m_X)^2}{N}$$

* Standard deviation (one-sigma)

$$\sigma_X := \sqrt{r_X}$$

# Combination of Scalar Random Variables

* Covariance $r_{XY} := E[(X - m_X)(Y - m_Y)] = \dfrac{\sum_{i=1}^{N}(x_i - m_X)(y_i - m_Y)}{N}$

* Given

$$Z = X + Y \qquad \left(m_X = 0, m_Y = 0, r_{XY} = 0\right)$$

then,

$$m_z = 0$$

$$r_z = r_X + r_Y$$

# Random Vector

* Given

$$V := \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \qquad X \sim (0, r_X), \qquad Y \sim (0, r_Y), \qquad Z \sim (0, r_Z)$$

then,

$$V \sim \left( O_{3\times1}, \begin{bmatrix} r_X & r_{XY} & r_{XZ} \\ r_{YX} & r_Y & r_{YZ} \\ r_{ZX} & r_{ZY} & r_Z \end{bmatrix} \right)$$

# Kalman Filtering

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# Dynamics Model

* state: minimum information to describe a system at a specific time instant

$$X_k, \quad X_{k+1}, \quad X_{k+2}, \quad \cdots$$

* dynamics model: a model that relates two states at different time instants

$$X_{k+1} = F_k X_k + w_k$$

* dynamics model can be constructed by
- given conditions: fixed position or trajectory
- experiences: a heavy vehicle endures small acceleration changes
- high speed sensors: odometr, gyro, accelerometer, compass

* a dynamics model acts as an equivalent measurement for incremental states


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# Gauss Markov Process

* A sequence of random vectors $\{X_k\}_{k=0,1,2,\cdots}$ driven by

a **dynamics model** $X_{k+1} = F_k X_k + w_k$

with a **Gaussian initial value** $X_0 \sim (0, P_0)$,

and a **white Gaussian noise sequence** $w_k \sim (0, q)$

is a **Gauss Markov process.**

* In this case, each $X_k$ satisfies the following properties.

$$X_k \sim (O, P_k), \qquad P_{k+1} = F_k P_k F_k^T + q$$

$$E[X_{k+1} X_k^T] = F_k P_k$$

# Measurement Model

* A stochastic measurement model is usually given as follows,

$$Y = H\,X + V$$

where

$Y$ : vector that consists of measured values

$H$ : observation matrix (mapping between the measured values and the state of interest)

$X$ : state vector (contains position, velocity, clock bias, Markov states describing sensor errors)

$V \sim (0, R)$: measurement noise (usually white Gaussian)

# Equivalent Measurements

* Sensor output  (state measurements)

$$Y_k = H_k X_k + V_k, \quad V_k \sim (0, R)$$

* *A priori* state estimate $\bar{X}_k$ at current time step provided by a filter

$$Y_k = X_k + \bar{V}_k, \quad \bar{V}_k \sim (0, \bar{P}_k)$$

* *A posteriori* state estimate $\hat{X}_{k-1}$ at previous time step

combined with a dynamics model $X_{k+1} = F_k X_k + W_k, \quad W_k \sim (0, Q)$

Construct

$$Y_k := F_{k-1} \hat{X}_{k-1},$$

where

$$\hat{X}_{k-1} = X_{k-1} + \delta X_{k-1}, \quad \delta X_{k-1} \sim (0, P_{k-1})$$

then

$$\begin{aligned} Y_k &= F_{k-1} X_{k-1} + F_{k-1} \delta X_{k-1} \\ &= (X_k - W_{k-1}) + F_{k-1} \delta X_{k-1} \\ &= X_k + \overline{V}_k \end{aligned}$$

where

$$\overline{V}_k = F_{k-1} \delta X_{k-1} - W_{k-1}$$

Thus,

$$Y_k = X_k + \overline{V}_k, \quad \overline{V}_k \sim (0, F_{k-1} \hat{P}_{k-1} F_{k-1}^T + Q)$$

# Direct Estimation

* Given

$$Y = H\,X + V, \qquad V \sim (O, R), \qquad \det(H^T H) \neq 0,$$

apply weighted pseudo inverse $H^+ := (H^T R^{-1} H)^{-1} H^T R^{-1}$ to obtain

- state estimate: $\hat{X} = H^+ Y = X + \hat{V}$

- estimation error: $\hat{V} = H^+ V \sim \left(O, (H^T R^{-1} H)^{-1}\right)$

- residual: $Z = H\hat{X} - Y = [HH^+ - I]V$

$$Z \sim (O, \Sigma)$$

$$\Sigma = [I - HH^+]R[I - HH^+]^T$$

# Indirect Estimation Given Initial Guess

* Given

$$Y = H\,X + V, \qquad V \sim \left(O, R\right), \qquad \det(H^T H) \neq 0,$$

with intial guess $\bar{X} \sim (X, \bar{P})$ (*a priori* information) such that

$$\bar{X} = X + \delta\bar{X},\;\; \delta\bar{X} \sim (O, \bar{P})$$

* Solution by augmented equivalent measurement vector:

$$\tilde{Y} = \begin{bmatrix} \bar{X} \\ Y \end{bmatrix} = \tilde{H}X + \tilde{V}, \quad \tilde{H} = \begin{bmatrix} I \\ H \end{bmatrix}, \quad \tilde{V} \sim \left(O, \tilde{R}\right), \quad \tilde{R} = \begin{bmatrix} \bar{P} & O \\ O & R \end{bmatrix}$$

$$\hat{X} = \tilde{H}^{+}\tilde{Y} = \bar{X} - K(H\bar{X} - Y) = \bar{X} - KZ$$

$$K = \bar{P}H^T(H\bar{P}H^T + R)^{-1} : Kalman\ gain$$

$$Z = H\bar{X} - Y : Indirect\ measurement$$

* Summary of indirect estimation

1. form indirect measurement $Z = H\overline{X} - Y \quad (Z = H\delta\overline{X} - V)$

2. compute Kalman gain $K = \overline{P}H^T(H\overline{P}H^T + R)^{-1}$

3. obtain improved estimate $\hat{X} = \overline{X} - KZ \quad (\delta\hat{X} = (I - KH)\delta\overline{X} + KV)$

4. then the improved estimate will satisfy

$$\hat{X} = X + \delta\hat{X}$$

$$\delta\hat{X} \sim \left(O, \hat{P}\right)$$

$$\hat{P} = (I - KH)\overline{P}(I - KH)^T + KRK^T$$

# Kalman Filter Algorithm

* **TIME PROPAGATION** by dynamics model $F_k$

:use dynamics model to project the estimate of previous time step

$$\overline{X}_{k+1} = F_k \hat{X}_k \sim (X_{k+1}, \overline{P}_{k+1})$$

$$\overline{P}_{k+1} = F_k \hat{P}_k F_k^T + Q_k$$

* **MEASUREMENT UPDATE** with newly-arrived measurement $Y_k$

:use indirect estimation with initial guess (*a priori* information)

$$Z_k = H_k \overline{X}_k - Y_k$$

$$K_k = \overline{P}_k H_k^T (H_k \overline{P}_k H_k^T + R_k)^{-1}$$

$$\hat{X}_k = \overline{X}_k - K_k Z_k \sim (X_k, \hat{P}_k)$$

$$\hat{P}_k = (I - K_k H_k) \overline{P}_k (I - K_k H_k)^T + K_k R_k K_k^T$$