

ETRI 원내 전문 교육

# GPS/관성센서 통합에 의한 측위 및 응용

## LEC3 DGPS FUNDAMENTALS

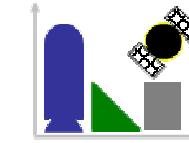
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항법 및 정보시스템 연구실  
Navigation & Information Systems Laboratory

항공전자 및 정보통신공학부

 **한국항공대학교**  
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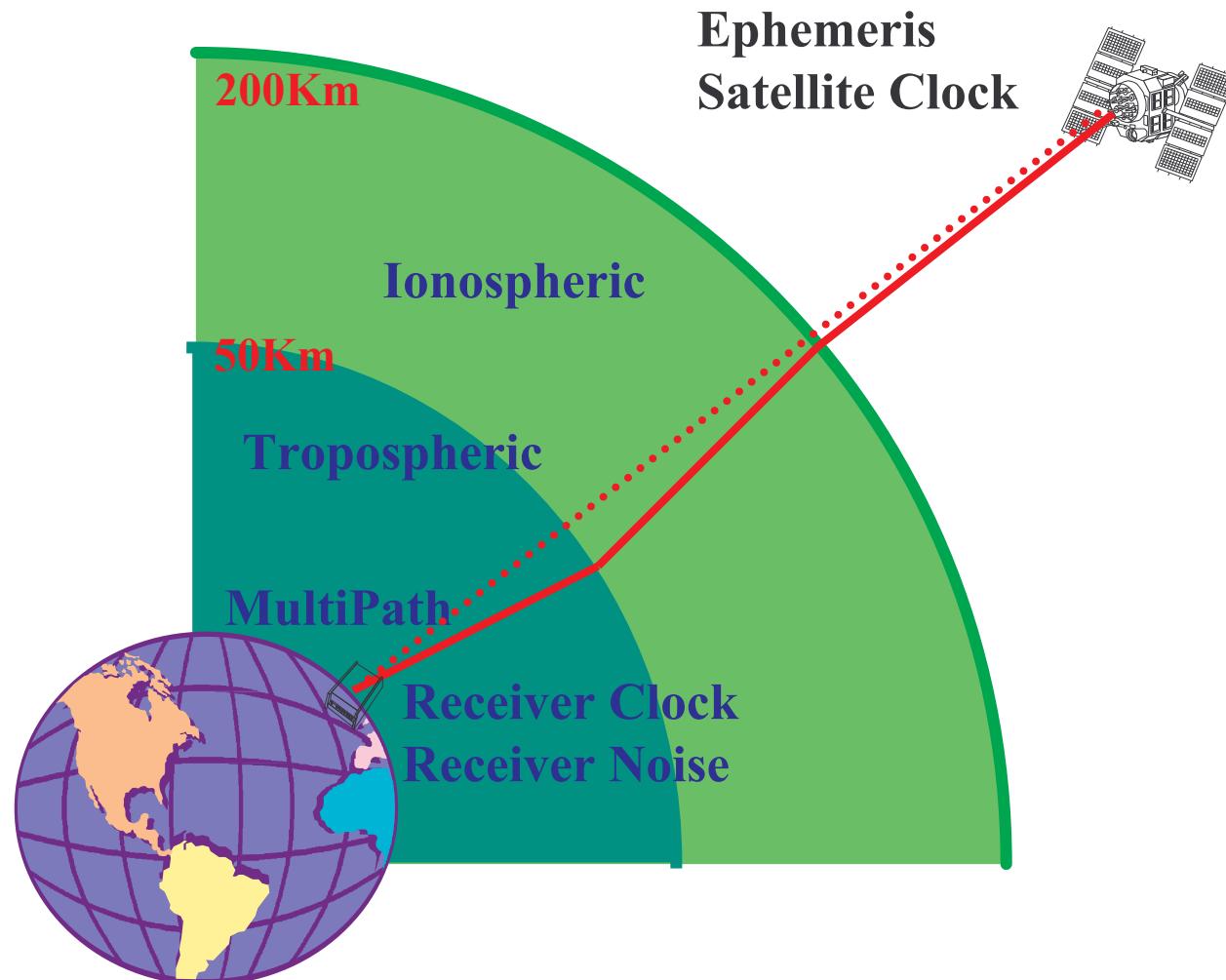
# Error Sources

방법 및 정보시스템 연구실  
Navigation & Information Systems Laboratory

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# 오차 요인



# 오차 크기

	표준 GPS	Differential GPS
위성 시계	1.5	0
궤도 오차	2.5	0
전리층	5.0	0.4
대류층	0.5	0.2
수신기 잡음	0.3	0.3
다중경로 오차	0.6	0.6
SA	30	0 (현재 영향 없음)

# *Dilution Of Precision (DOP)*

- 관측행렬  $H$ 에 나타난 위성 시선각 분포의 영향
- 오차공분산 행렬  $\sigma_p (H^T H)^{-1}$  대각성분의 조합

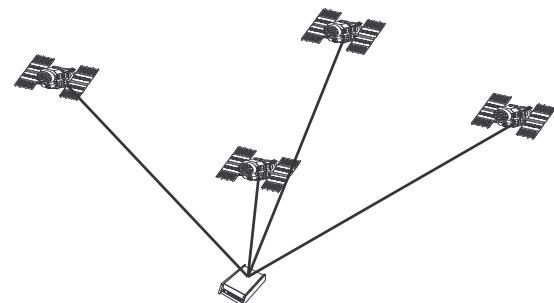
$$PDOP = \sqrt{\sigma_n^2 + \sigma_e^2 + \sigma_h^2}$$

$$VDOP = \sigma_h$$

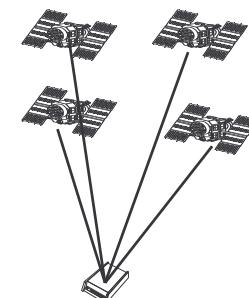
$$GDOP = \sqrt{\sigma_n^2 + \sigma_e^2 + \sigma_h^2 + \sigma_t^2 c^2}$$

$$HDOP = \sqrt{\sigma_n^2 + \sigma_e^2}$$

$$TDOP = \sigma_t$$



Good DOP



Bad DOP

# 시계 오차

- (위성 시계 오차) = (정밀한 시계와 broadcast 시계 사이의 오차)
- 위성 시계 오차는 1.5 m 거리 오차에 해당하는 약  $5 \times 10^{-9}$ 초
- 수신기 시계 오차는 위치오차와 함께 추정의 대상임
- 단독 수신기를 이용하여 수신기 시계를 보정하면 수신기 시계오차는 10 m 정도의 거리오차에 해당하는  $3.33 \times 10^{-8}$ 초
- first-order 시계오차는 차분 기술에 의해 제거가능

# 전리층 지연

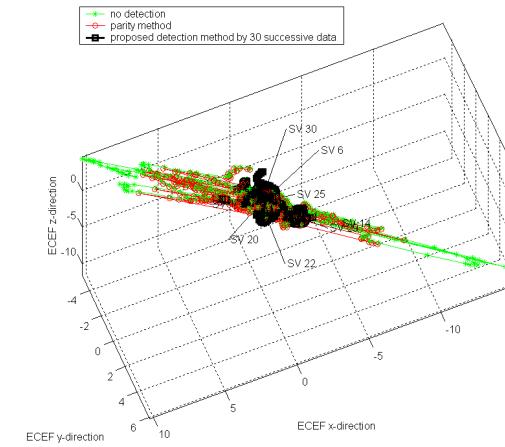
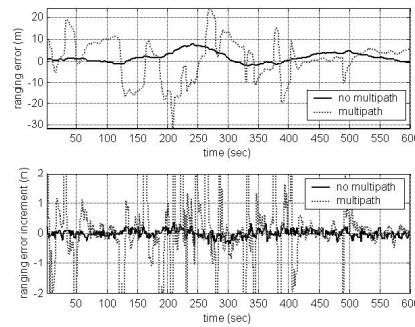
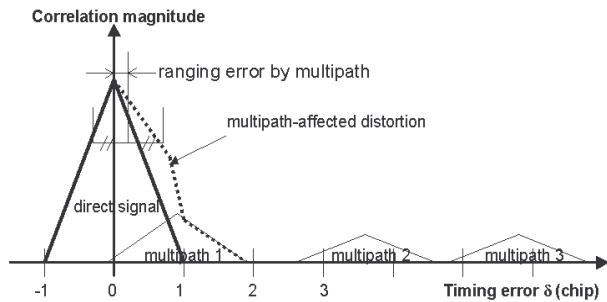
- 전리층에 존재하는 자유전자에 의하여 이를 통과하는 전 자기파는 분산 및 왜곡
- SA 제거 이후 가장 큰 오차 요인
- 의사거리는 더 길게 측정됨
- 위상은 더 짧게 측정됨
- 위상의 경우 주파수 제곱에 반비례
- 이중 주파수 수신기에 의하여 제거 가능

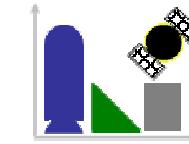
# 대류층 자연

- 앙각(elevation angle)에 영향을 받음
- 천정에서는 2.5m에 달하며, 5도 앙각에서는 20-28m 이상에 달할 수 있음
- 주로 경험적 모델(온도, 압력, 상대 습도의 함수)을 사용 하며, 이를 통하여 대부분의 효과를 추정할 수 있다
- 이중 주파수에 의한 제거는 불가능

# 다중경로 오차

- 안테나 주위의 물체와 들어오는 신호의 상호 작용의 결과로 복합적인 반사와 회절의 원인
- 의사거리 측정치의 경우 수십 m 정도의 오차 발생 가능
- 안테나의 종류 및 위치 선정에 의하여 영향
- 낮은 앙각 위성 신호에 포함될 가능성 높음 (**choke ring** 안테나 사용)
- 수신기 상관기 및 신호처리 알고리즘에 의한 완화 가능
- 완전히 제거하기 어려움





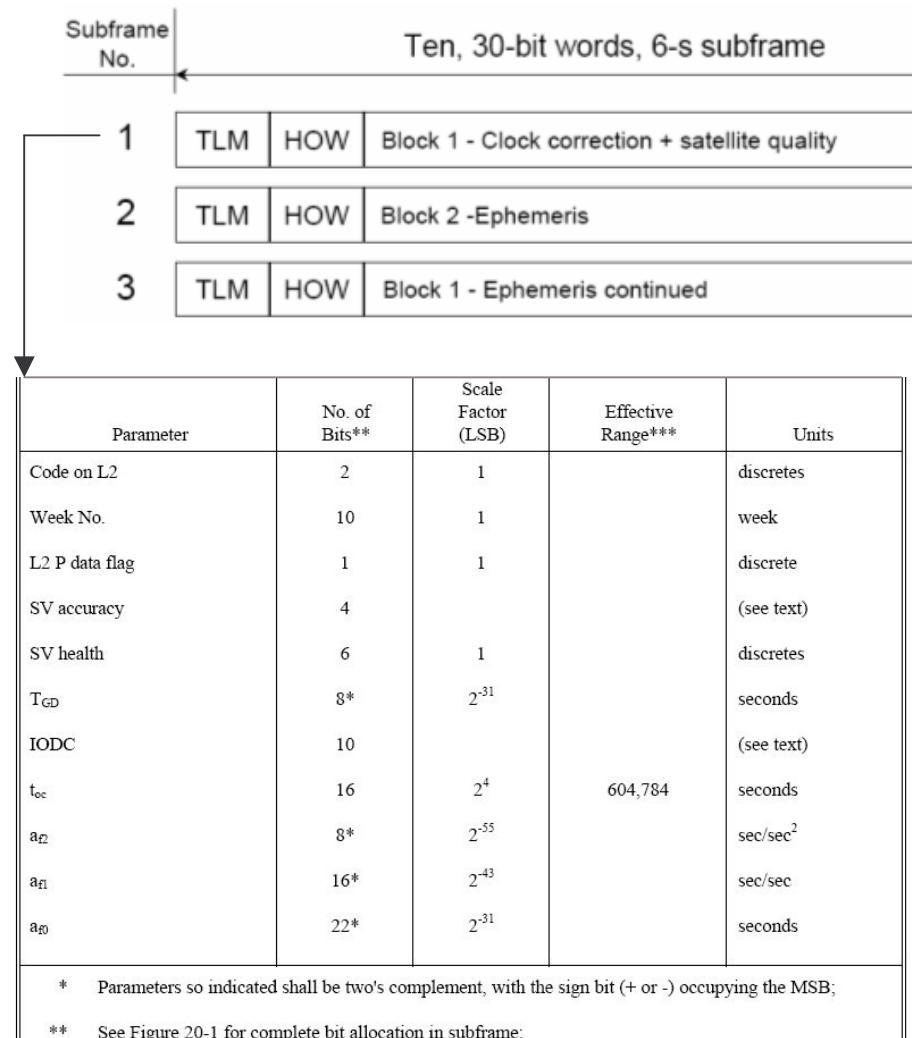
# Satellite Position & Correction Terms

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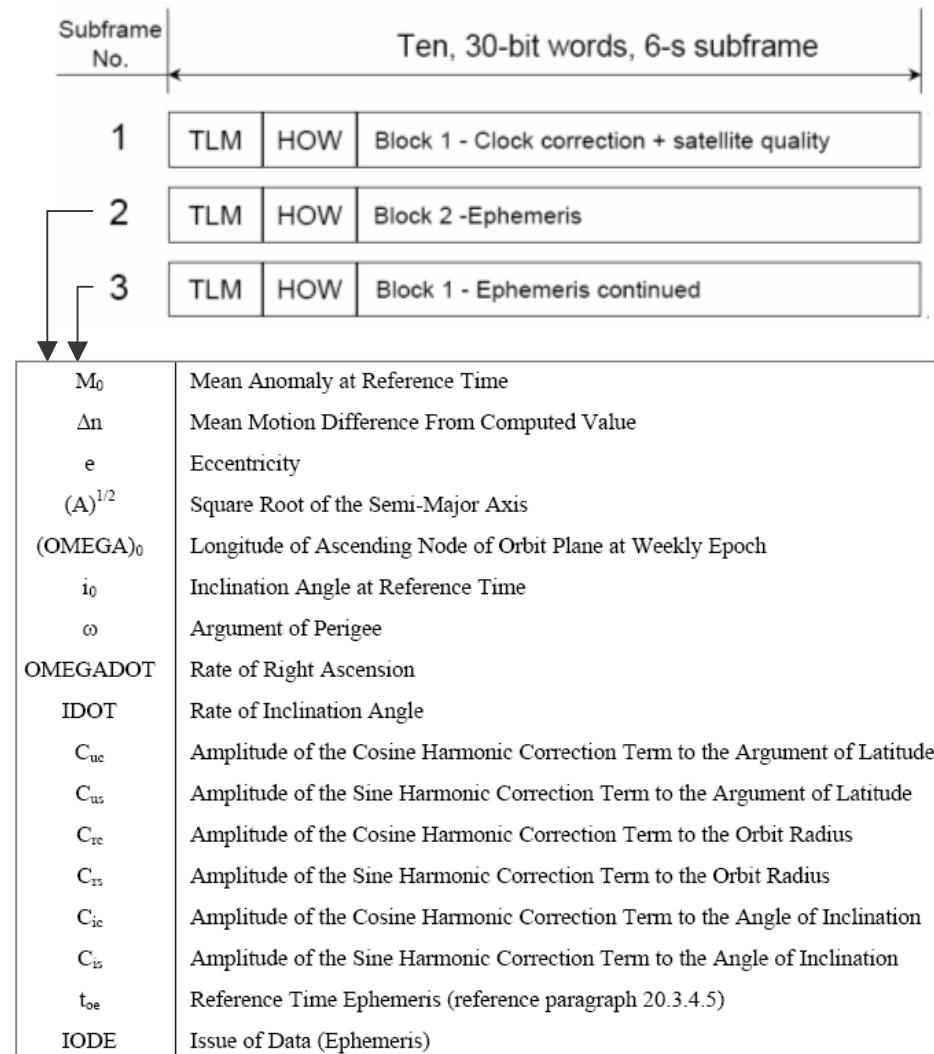
항공전자 및 정보통신공학부

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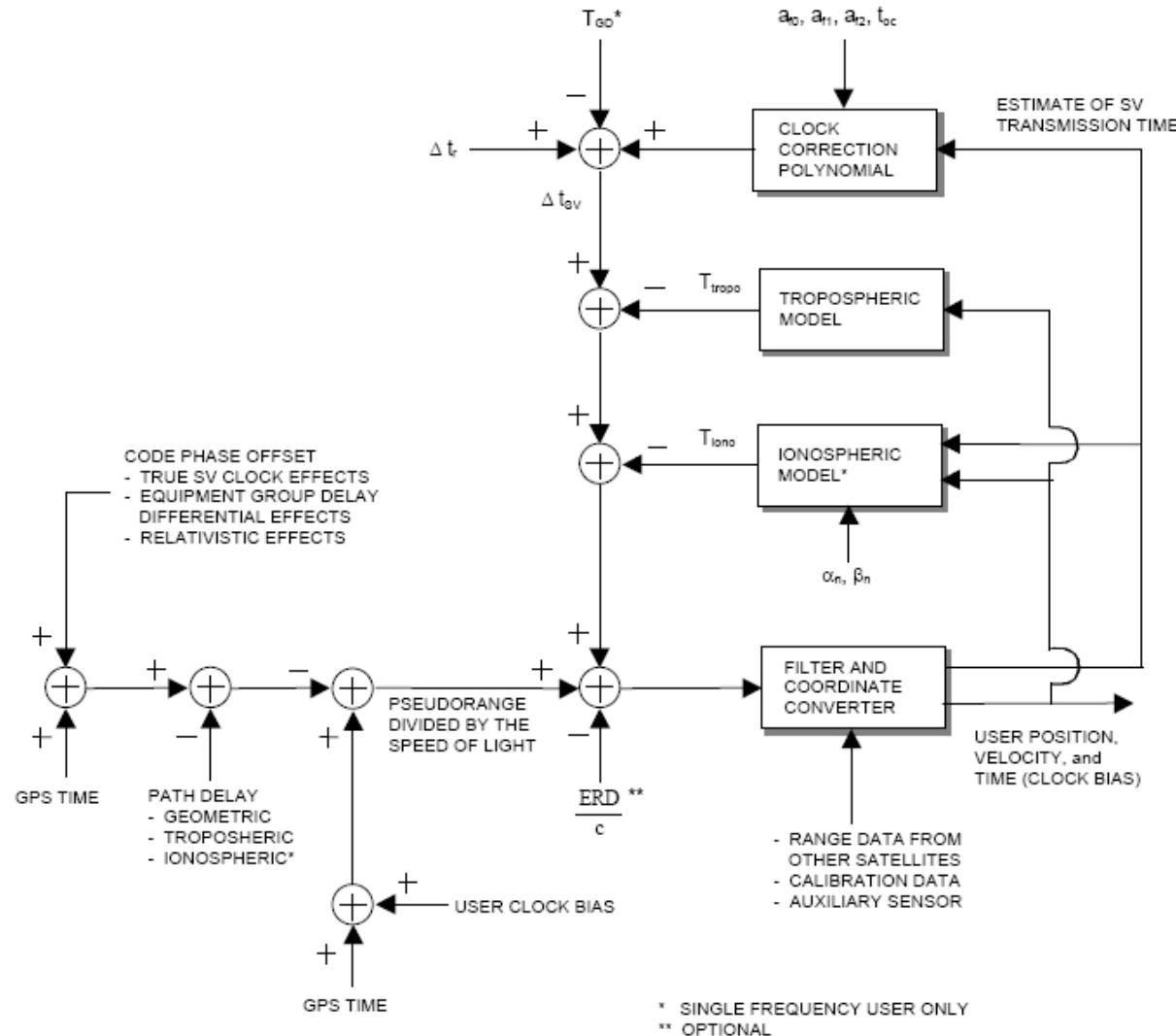
# Clock parameters in Navigation Message



# Orbit parameters in Navigation Message



# Computational Flow



# Satellite Clock Correction

$$\Delta t_{sv} = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2 + \Delta t_r$$

where

$t$  = GPS system time (seconds) =  $t_{sv} - \Delta t_{sv}$

$t_{sv}$  = effective SV PRN code phase time at message transmission time (seconds)

$\Delta t_{sv}$  = SV PRN code phase time offset (seconds)

$a_{f0}$ ,  $a_{f1}$  and  $a_{f2}$  are the polynomial coefficients given in subframe 1

$t_{oc}$  is the clock data reference time

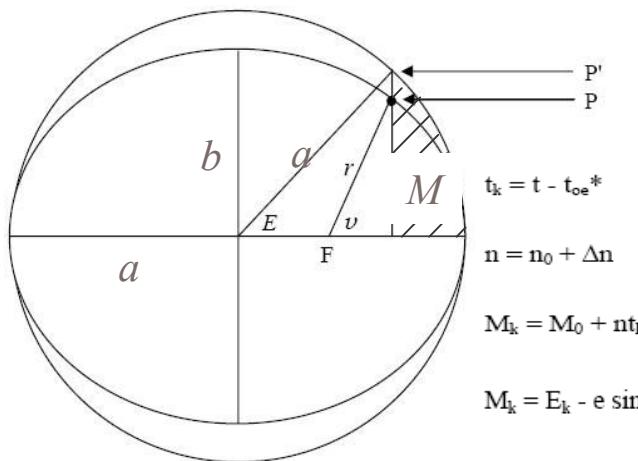
$$\Delta t_r = F e(A)^{1/2} \sin E_k$$

$$F = \frac{-2\mu}{c^2} = -4.442807633 (10)^{-10} \text{ sec/(meter)}^{1/2}$$

( $e$ ,  $A$ ,  $E_k$ ) ephemeris parameters

# Satellite Position 1: Mean, Eccentric, and True Anomalies

$$\underline{M \rightarrow E \rightarrow v \rightarrow \Phi} \rightarrow \{u, r, i\} \rightarrow \{x', y', z'\} \rightarrow \{x, y, z\}$$



Time from ephemeris reference epoch

Corrected mean motion

Mean anomaly

Kepler's Equation for Eccentric Anomaly  
(may be solved by iteration)(radians)

True Anomaly

$$v_k = \tan^{-1} \left\{ \frac{\sin v_k}{\cos v_k} \right\}$$

$$= \tan^{-1} \left\{ \frac{\sqrt{1-e^2} \sin E_k / (1-e \cos E_k)}{(\cos E_k - e) / (1-e \cos E_k)} \right\}$$

$$E_k = \cos^{-1} \left\{ \frac{e + e \cos v_k}{1 + e \cos v_k} \right\}$$

Eccentric Anomaly

## Satellite Position 2:

### 위성좌표계에서 ECEF좌표계로의 변환

$$M \rightarrow E \rightarrow V \rightarrow \Phi \rightarrow \{u, r, i\} \rightarrow \{x', y', z'\} \rightarrow \{x, y, z\}$$

위성좌표계

ECEF좌표계

$$\Phi_k = v_k + \omega$$

$$\begin{aligned}\delta u_k &= c_{us} \sin 2\Phi_k + c_{uc} \cos 2\Phi_k \\ \delta r_k &= c_{rs} \sin 2\Phi_k + c_{rc} \cos 2\Phi_k \\ \delta i_k &= c_{is} \sin 2\Phi_k + c_{ic} \cos 2\Phi_k\end{aligned}$$

$$u_k = \Phi_k + \delta u_k$$

$$r_k = A(1 - e \cos E_k) + \delta r_k$$

$$i_k = i_0 + \delta i_k + (\text{IDOT}) t_k$$

$$\left. \begin{aligned}x'_k &= r_k \cos u_k \\ y'_k &= r_k \sin u_k\end{aligned}\right\}$$

$$\Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e) t_k - \dot{\Omega}_e t_{ee}$$

$$\left. \begin{aligned}x_k &= x'_k \cos \Omega_k - y'_k \cos i_k \sin \Omega_k \\ y_k &= x'_k \sin \Omega_k + y'_k \cos i_k \cos \Omega_k \\ z_k &= y'_k \sin i_k\end{aligned}\right\}$$

Argument of Latitude Correction  
Radius Correction  
Inclination Correction

Argument of Latitude

Second Harmonic Perturbations

Corrected Argument of Latitude

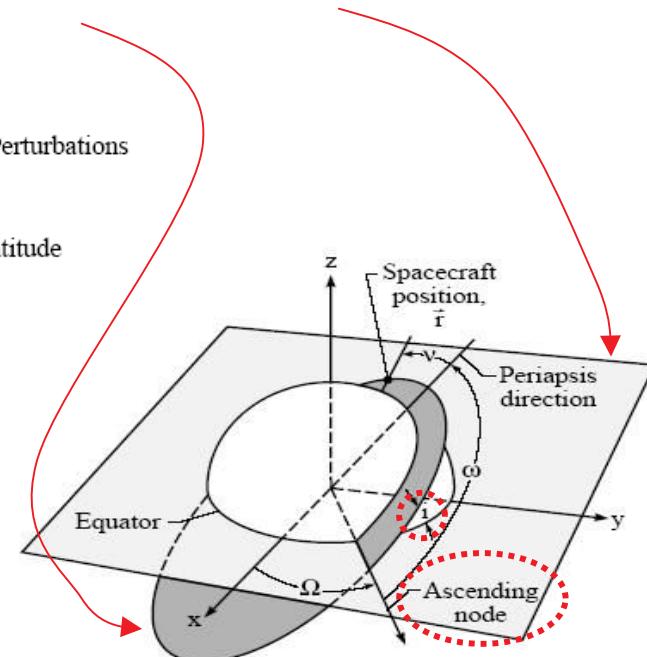
Corrected Radius

Corrected Inclination

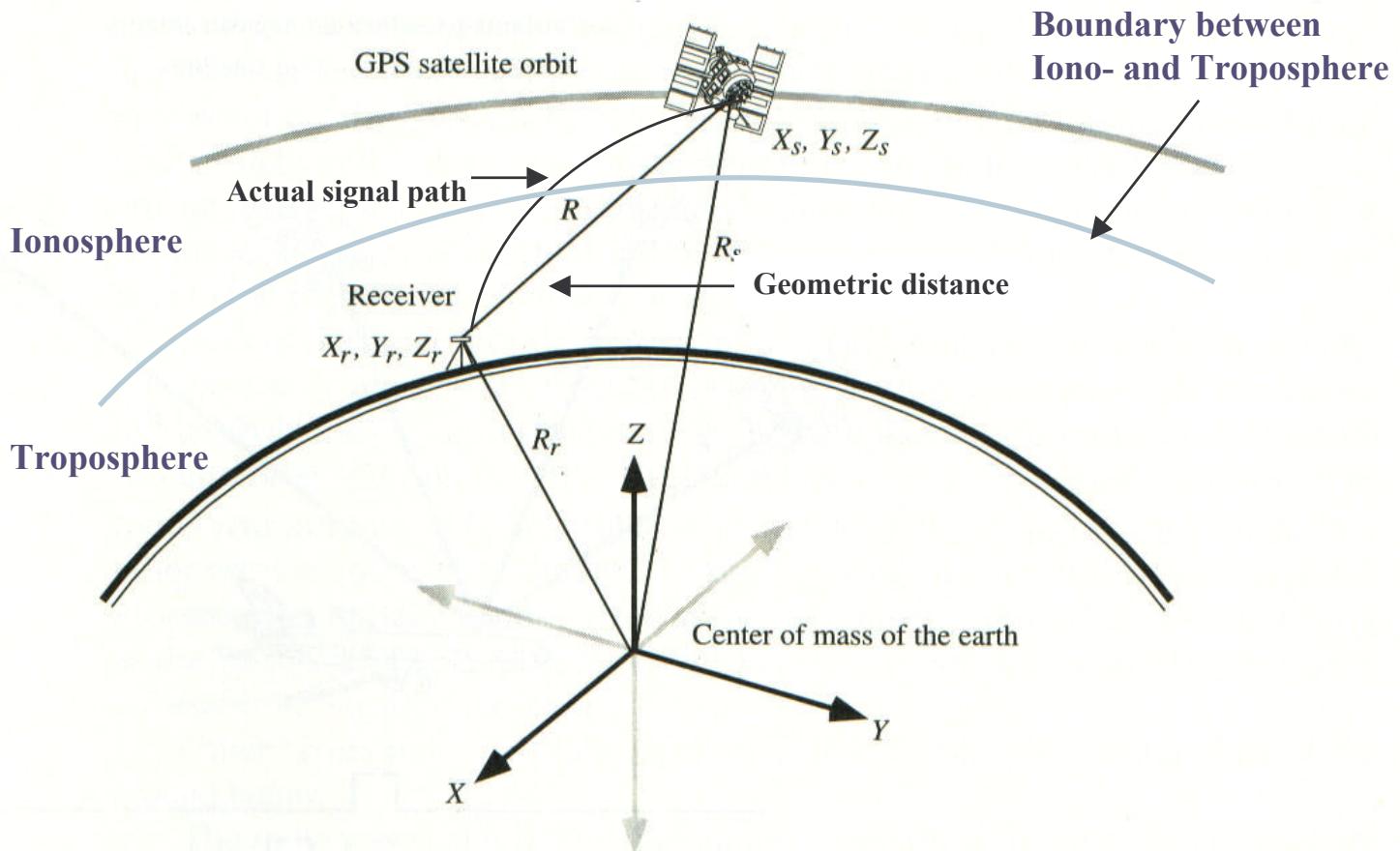
Positions in orbital plane.

Corrected longitude of ascending node.

Earth-fixed coordinates.



# 전리층 및 대류층 지역



# 전리층 보정 (*Klobuchar model, 단일주파수 수신기*)

$\alpha_n, \beta_n, n = 0, 1, 2, \text{and } 3$  (ion. param.s)  
 $\lambda_u$  (longitude)  
 $\phi_u$  (latitude)  
GPS time



$$T_{\text{iono}} = \begin{cases} F * \left[ 5.0 * 10^{-9} + (\text{AMP}) \left( 1 - \frac{x^2}{2} + \frac{x^4}{24} \right) \right], |x| < 1.57 \\ F * (5.0 * 10^{-9}) , |x| \geq 1.57 \end{cases} \text{ (sec)}$$

where—

$$\text{AMP} = \begin{cases} \sum_{n=0}^3 \alpha_n \phi_m^n, \text{AMP} \geq 0 \\ \text{if AMP} < 0, \text{AMP} = 0 \end{cases} \text{ (sec)}$$

$$\phi_m = \phi_i + 0.064 \cos(\lambda_i - 1.617) \quad \text{(semi-circles)}$$

$$\lambda_i = \lambda_u + \frac{\psi \sin A}{\cos \phi_i} \quad \text{(semi-circles)}$$

$$x = \frac{2\pi (t - 50400)}{\text{PER}} \quad \text{(radians)}$$

$$\phi_i = \begin{cases} \phi_u + \psi \cos A \text{ (semi-circles), } |\phi_i| \leq 0.416 \\ \text{if } \phi_i > +0.416, \text{ then } \phi_i = +0.416 \\ \text{if } \phi_i < -0.416, \text{ then } \phi_i = -0.416 \end{cases} \quad \text{(semi-circles)}$$

$$\text{PER} = \begin{cases} \sum_{n=0}^3 \beta_n \phi_m^n, \text{PER} \geq 72,000 \\ \text{if PER} < 72,000, \text{PER} = 72,000 \end{cases} \text{ (sec)}$$

$$\psi = \frac{0.0137}{E + 0.11} - 0.022 \quad \text{(semi-circles)}$$

$$F = 1.0 + 16.0 [0.53 - E]^3$$

$$t = 4.32 * 10^4 \lambda_i + \text{GPS time} \quad \text{(sec)}$$

$\alpha_n$  and  $\beta_n$  satellite transmitted data words with  $n = 0, 1, 2, \text{and } 3$

$0 \leq t < 86400$ : therefore, if  $t \geq 86400$  seconds, subtract 86400 seconds;

if  $t < 0$  seconds, add 86400 seconds.

# 대류층 보정 (*Chao, Magnavox, Collins Model*)

## ● Chao Model

$$\Delta\rho_{trop} = \Delta\rho_{dry} F_{dry} + \Delta\rho_{wet} F_{wet}$$

$$\Delta\rho_{dry} = 2.276 \times 10^{-5} P$$

$$F_{dry} = \left[ \sin(E) + \frac{0.00143}{\tan(E) + 0.0445} \right]^{-1}$$

$$\Delta\rho_{wet} = 4.70 \times 10^2 \frac{e^{1.23}}{T^2} + 1.705 \times 10^6 \alpha \frac{e^{1.46}}{T^3}$$

$$F_{wet} = \left[ \sin(E) + \frac{0.00035}{\tan(E) + 0.017} \right]^{-1}$$

## ● Magnavox Model

$$\Delta\rho_M = \frac{2.208}{\sin(E)} \left[ e^{-\frac{h_r}{6900}} - e^{-\frac{h_s}{6900}} \right]$$

## ● Collins Model

$$\Delta\rho_c = \frac{2.4225}{0.026 + \sin(E)} e^{-\frac{h_r}{7492.8}}$$

$P$  : pressure (newton/m<sup>2</sup>)

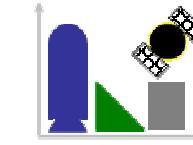
$T$  : temperature (degrees Kelvin)

$e$  : partial pressure of water vapor (millibars)

$E$  : satellite elevation (deg)

$h_r$  : receiver altitude (m)

$h_s$  : satellite altitude (m)



# Standalone Positioning

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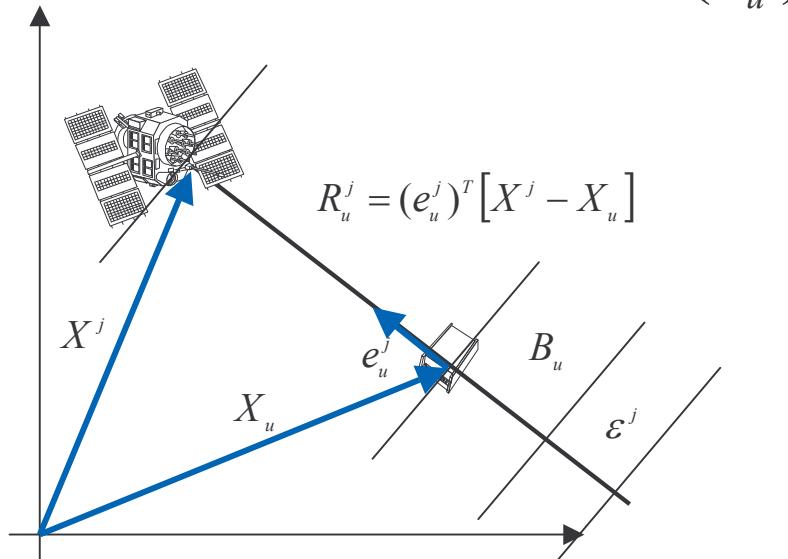


# 각 채널별 의사거리 측정치

1. Measure PR  $\tilde{\rho}_u^j = R_u^j + B_u - B^j + I^j + T^j + W^j$

$$= R_u^j + B_u + \varepsilon^j$$

$$= (e_u^j)^T [X^j - X_u] + B_u + \varepsilon^j$$



$$e_u^j = \frac{X^j - X_u}{\sqrt{(X^j - X_u)^T (X^j - X_u)}} \text{ nonlinear!} \\ \rightarrow \text{linearize}$$

$\tilde{\rho}_u^j$  : measured pseudorange

$R_u^j$  : true range

$B_u$  : user clock bias

$B^j$  : satellite clock bias

$I^j$  : ionospheric delay

$T^j$  : tropospheric delay

$W^j$  : thermal noise

$\varepsilon^j$  : total pseudorange measurement error

$X_u$  : user position

$e_u^j (\|e_u^j\|=1)$  : LOS vector between user and satellite

# 각 채널별 의사거리 추정치

Given

$$\hat{X}_u = X_u + \delta X_u$$

$$\hat{B}_u = B_u + \delta B_u$$

## 2. Compute estimated PR as

$$\hat{\rho}_u^j = (\hat{e}_u^j)^T [X^j - \hat{X}_u] + \hat{B}_u$$

$\hat{\rho}_u^j$  : estimated pseudorange

$\hat{X}_u$  : estimated user position

$\delta X_u$  : position (estimation) error

$\hat{B}_u$  : estimated user clock bias

$\delta B_u$  : clock bias (estimation) error

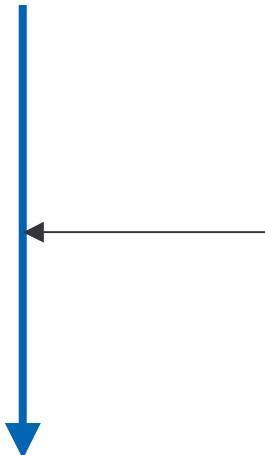
$\hat{e}_u^j (\| \hat{e}_u^j \| = 1)$  : estimated LOS vector based on  $\hat{X}_u$

# 선형화된 채널별 의사거리 간접 측정치

## 3. Subtract estimated PR from Measured PR

$$z^j = \tilde{\rho}_u^j - \hat{\rho}_u^j$$

Since



$$\begin{aligned} z^j &= (e_u^j)^T [X^j - X_u] + B_u + \varepsilon^j \\ &\quad - (e_u^j)^T [X^j - \hat{X}_u] - \hat{B}_u \\ &= (e_u^j)^T \delta X_u - \delta B_u + \varepsilon^j \\ &= [(e_u^j)^T \cdots - 1] \begin{bmatrix} \delta X_u \\ \dots \\ \delta B_u \end{bmatrix} + \varepsilon^j \end{aligned}$$

It satisfies

$$\begin{aligned} z^j &= h^j \begin{bmatrix} \delta X_u \\ \dots \\ \delta B_u \end{bmatrix} - \varepsilon^j \\ h^j &= [(e_u^j)^T \cdots - 1] \end{aligned}$$

# 선형화된 벡터 간접 측정치

## 4. Stack (at least 4) scalar measurements into a vector

$$Z = \begin{bmatrix} z^1 \\ z^2 \\ \vdots \\ z^m \end{bmatrix}^j \quad H = \begin{bmatrix} h^1 \\ h^2 \\ \vdots \\ h^m \end{bmatrix}^j$$

The vector measurement satisfies

$$Z = H \begin{bmatrix} \delta X_u \\ \dots \\ \delta B_u \end{bmatrix} - \varepsilon$$

$$\varepsilon = \begin{bmatrix} \varepsilon^1 \\ \varepsilon^2 \\ \vdots \\ \varepsilon^m \end{bmatrix}^j$$

# 위치오차 및 시계오차의 계산

## 5. Multiply pseudoinverse to the vector measurement

$$(H^T H)^{-1} H^T \rightarrow Z$$

## 6. Then, estimates of position and clock bias error are obtained

$$\begin{bmatrix} \hat{\delta X}_u \\ \hat{\delta B}_u \end{bmatrix} = (H^T H)^{-1} H^T Z$$
$$= \begin{bmatrix} \delta X_u \\ \delta B_u \end{bmatrix} - \underbrace{(H^T H)^{-1} H^T \varepsilon}_{\text{estimation error}}$$

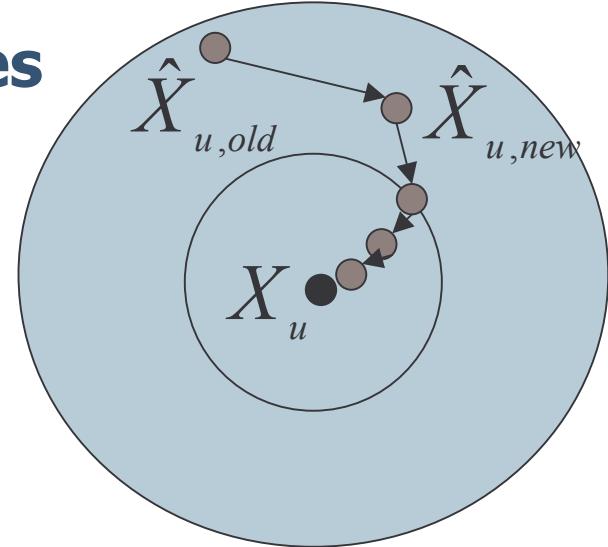
# 위치 및 시계오차의 보상

## 7. Update old estimates to new estimates

$$\hat{X}_{u,new} = \hat{X}_{u,old} - \delta \hat{X}_u$$

$$\hat{B}_{u,new} = \hat{B}_{u,old} - \delta \hat{B}_u$$

## 8. Iterate 2~7 until converges

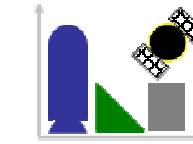


\* If 8 is finished, the following condition is satisfied

$$\begin{bmatrix} \delta\hat{X}_u \\ \delta\hat{B}_u \end{bmatrix} \rightarrow O_{4 \times 1} \Rightarrow \begin{bmatrix} \delta X_u \\ \delta B_u \end{bmatrix} \rightarrow (H^T H)^{-1} H^T \varepsilon \\ = (H^T H)^{-1} H^T (-B + I + T + W)$$

*Estimation error*      *Dilution Of Precision (DOP)*      *SV clock error*      *iono error*      *tropo error*      *noise*

For, more detailed information, see "error sources"



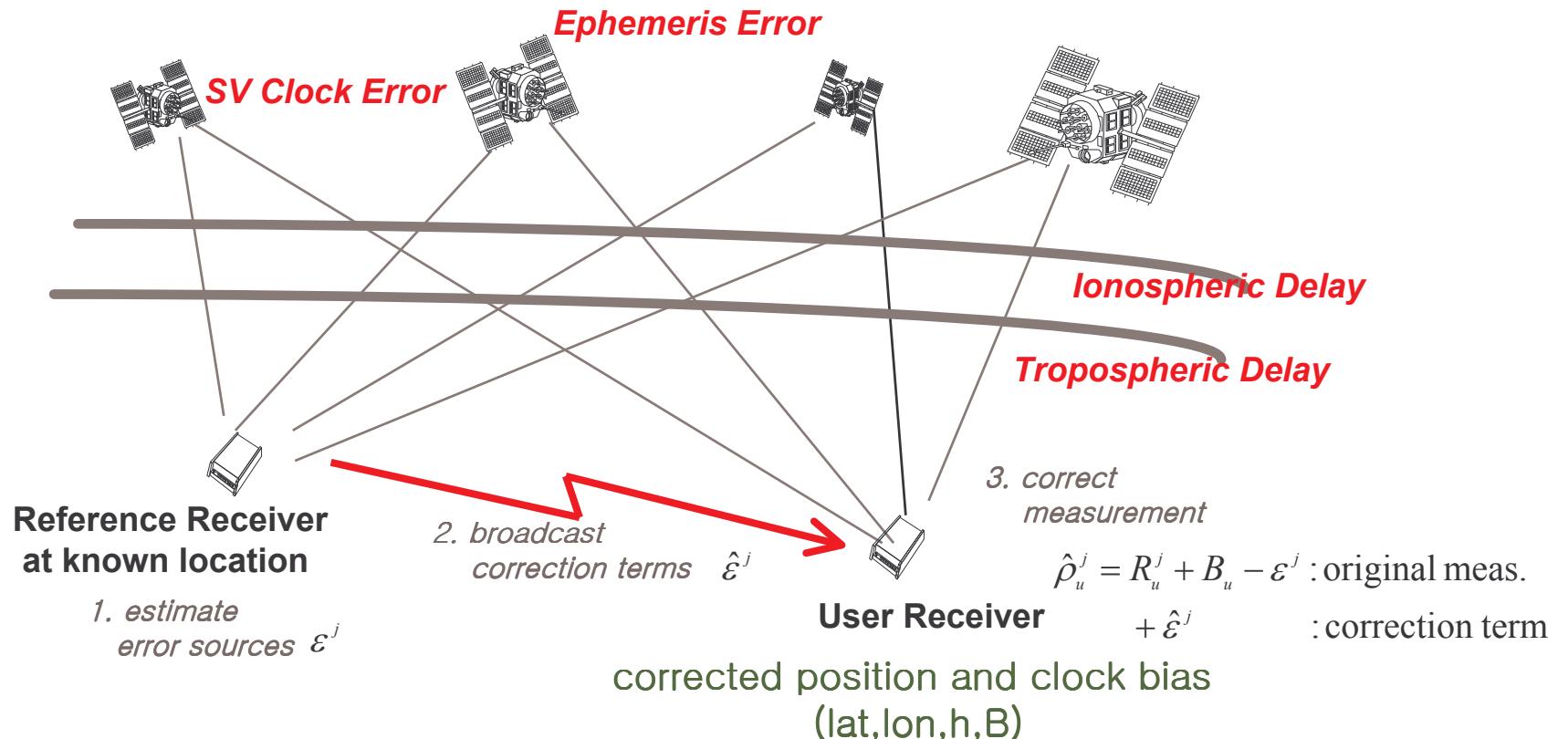
# Differential Positioning

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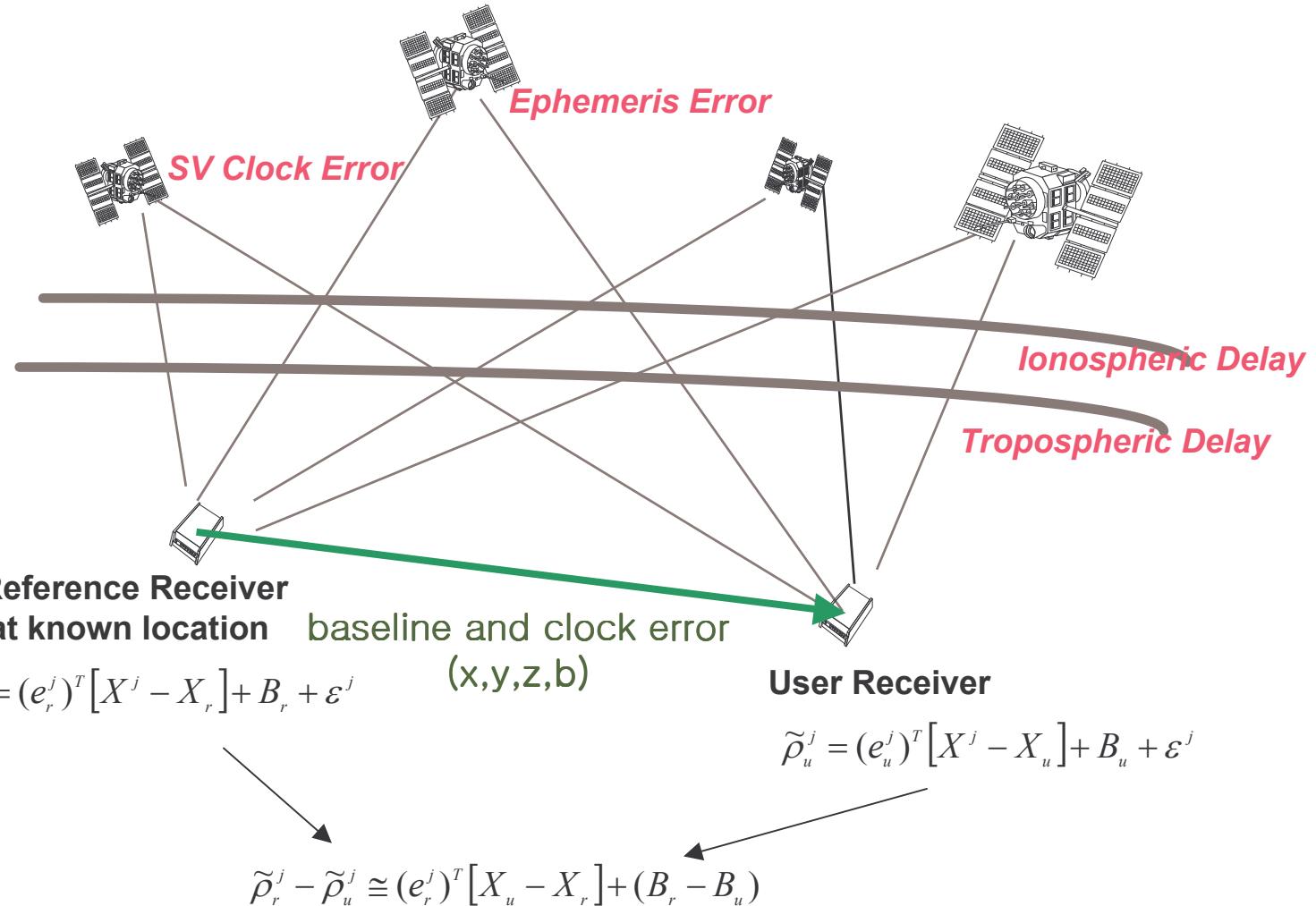
항공전자 및 정보통신공학부



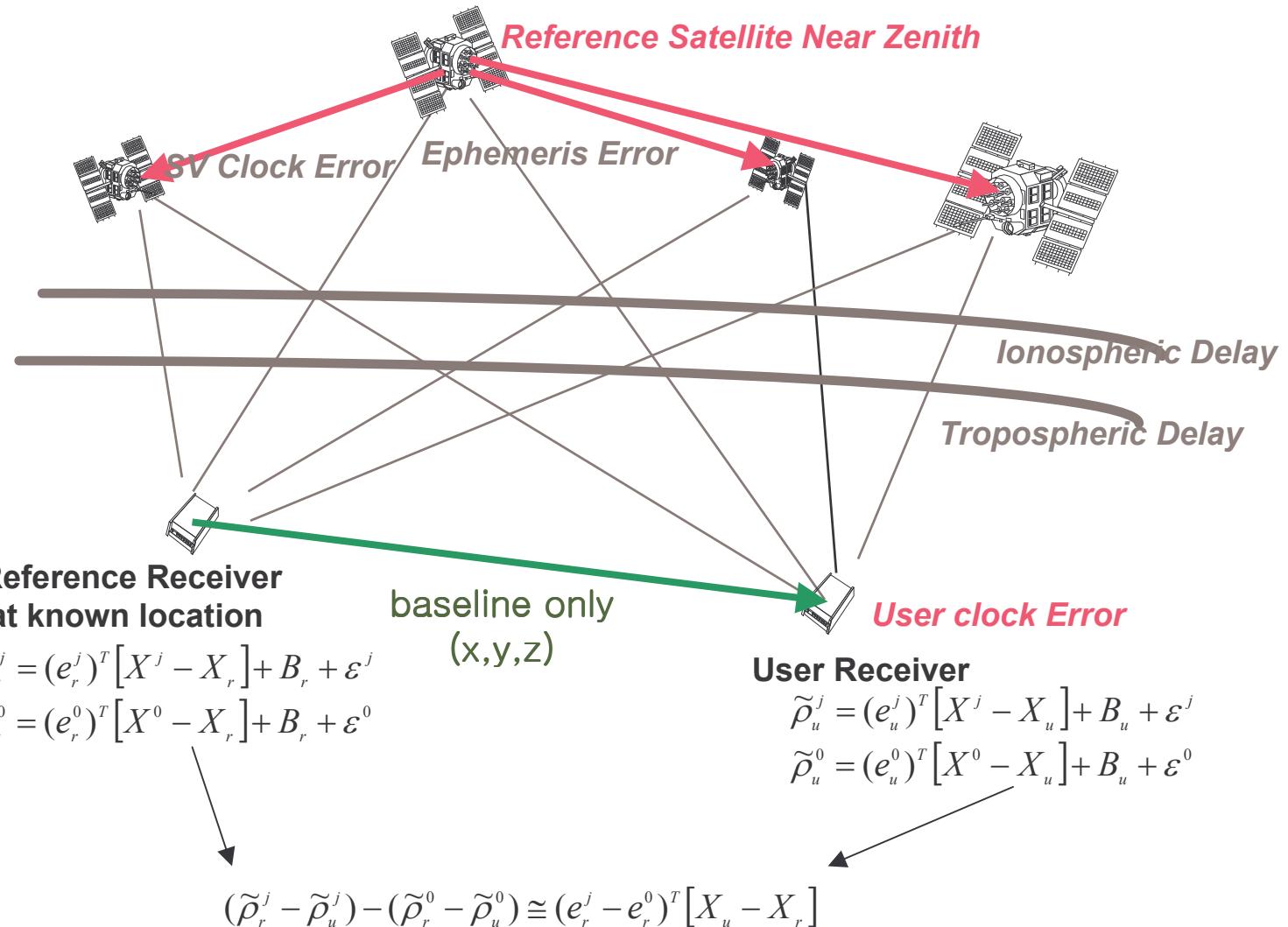
# 오차 추정 및 방송 (*Estimation and Broadcasting*)



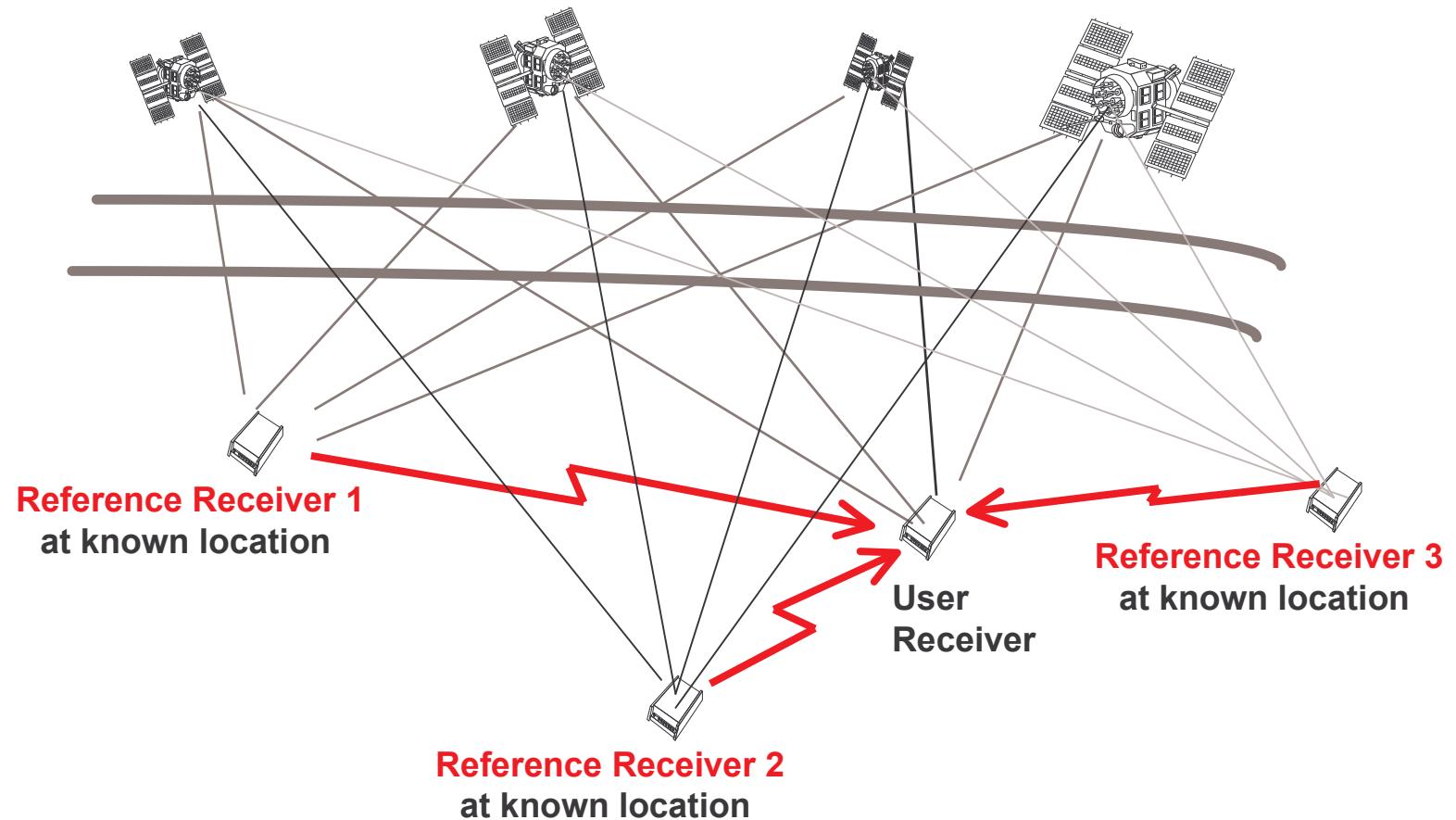
# 단일 차분(Single Difference)



# 이중 차분(Double Difference)



# DGPS망(Network)



## ***Single-Differenced (SD) GPS Measurement Model***

$$\text{PR: } \tilde{\rho}_{j,k} = e_{j,k}^T (x_{j,k} - x_{u,k}) + b_{u,k} + v_{j,k}$$

$$\text{CR: } \tilde{\phi}_{j,k} = e_{j,k}^T (x_{j,k} - x_{u,k}) + b_{u,k} + n_{j,k} + \lambda N_j$$

where

$e_{j,k}$  : Line Of Sight (LOS) vector from the receiver to the j-th satellite

$x_{j,k}$  : Earth-Centered Earth-Fixed (ECEF) position of the j-th satellite

$x_{u,k}$  : ECEF receiver position

$b_{u,k}$  : receiver clock bias

$N_j$  : unresolved SD integer ambiguity

$v_{j,k}$  : SD code noise

$n_{j,k}$  : SD carrier noise

$$\begin{bmatrix} v_{j,k} \\ v_{j,k+1} \\ v_{j+1,k} \\ n_{j,k} \\ n_{j,k+1} \\ n_{j+1,k} \end{bmatrix} \sim \left( \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} r_\rho & 0 & 0 & 0 & 0 & 0 \\ 0 & r_\rho & 0 & 0 & 0 & 0 \\ 0 & 0 & r_\rho & 0 & 0 & 0 \\ 0 & 0 & 0 & r_\Phi & 0 & 0 \\ 0 & 0 & 0 & 0 & r_\Phi & 0 \\ 0 & 0 & 0 & 0 & 0 & r_\Phi \end{bmatrix} \right)$$

where

$r_\rho$  : code noise strength (uniform)

$r_\phi$  : carrier noise strength (uniform)

## ***Double-Differenced (DD) GPS Measurement Model***

$$\text{PR: } \tilde{\rho}_{j,k} = e_{j,k}^T (x_{u,k} - x_{r,k}) + v'_{j,k}$$

$$\text{CR: } \tilde{\phi}_{j,k} = e_{j,k}^T (x_{u,k} - x_{r,k}) + n'_{j,k} + \lambda N'_j$$

where

$e_{j,k}$  : Line Of Sight (LOS) vector from the receiver to the j-th satellite

$x_{r,k}$  : ECEF reference (fixed) receiver position

$x_{u,k}$  : ECEF user (moving) receiver position

$N'_j$  : unresolved DD integer ambiguity

$v'_{j,k}$  : DD code noise

$n'_{j,k}$  : DD carrier noise

# 0/중 주파수 위상 측정치 조합

- 측정치 조합의 형태

$$\tilde{\Phi}_u^j(c1, c2) = c1 \tilde{\Phi}_u^j(L1) + c2 \tilde{\Phi}_u^j(L2)$$

where

$$\tilde{\Phi}_u^j(L1) = R_u^j + B_u - B^j \boxed{-I^j(L1)} + T^j + \lambda_{L1} N_{L1} + n^j(L1)$$

$$\tilde{\Phi}_u^j(L2) = R_u^j + B_u - B^j \boxed{-I^j(L2)} + T^j + \lambda_{L2} N_{L2} + n^j(L2)$$

$$I^j(L2) = \alpha I^j(L1),$$

$$\alpha = \frac{f_{L1}^2}{f_{L2}^2}$$

## ● 측정치 조합의 목적

- 전리층 오차 제거: iono-free combination

$$\tilde{\Phi}_u^j \left( \frac{\alpha}{\alpha-1}, \frac{-1}{\alpha-1} \right) = R_u^j + B_u - B^j + T^j + \lambda_{(c1,c2)} N_{(c1,c2)} + n^j(c1,c2)$$

*non-integer*

$$I^j(c1,c2) = 0$$

- 미지정수 결정 편의를 위한 파장 길이의 확장: widelane combination

$$\tilde{\Phi}_u^j \left( \frac{\lambda_{L1}}{\lambda_{L1} - \lambda_{L2}}, \frac{-\lambda_{L2}}{\lambda_{L1} - \lambda_{L2}} \right) = R_u^j + B_u - B^j - I^j(c1,c2) + T^j + \lambda_{(c1,c2)} N_{(c1,c2)} + n^j(c1,c2)$$

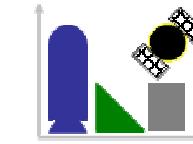
$$\lambda_{(c1,c2)} \cong 4\lambda_{L1}$$

- 전리층 오차에 의한 필터의 발산 방지: divergence-free combination

$$\tilde{\Phi}_u^j(2\alpha - 1, 2 - 2\alpha) = R_u^j + B_u - B^j + I^j + T^j + \lambda_{(c1,c2)} N_{(c1,c2)} + n^j(c1,c2)$$

*non-integer*

$$\tilde{\rho}_u^j = R_u^j + B_u - B^j + I^j + T^j + W^j(c1,c2)$$



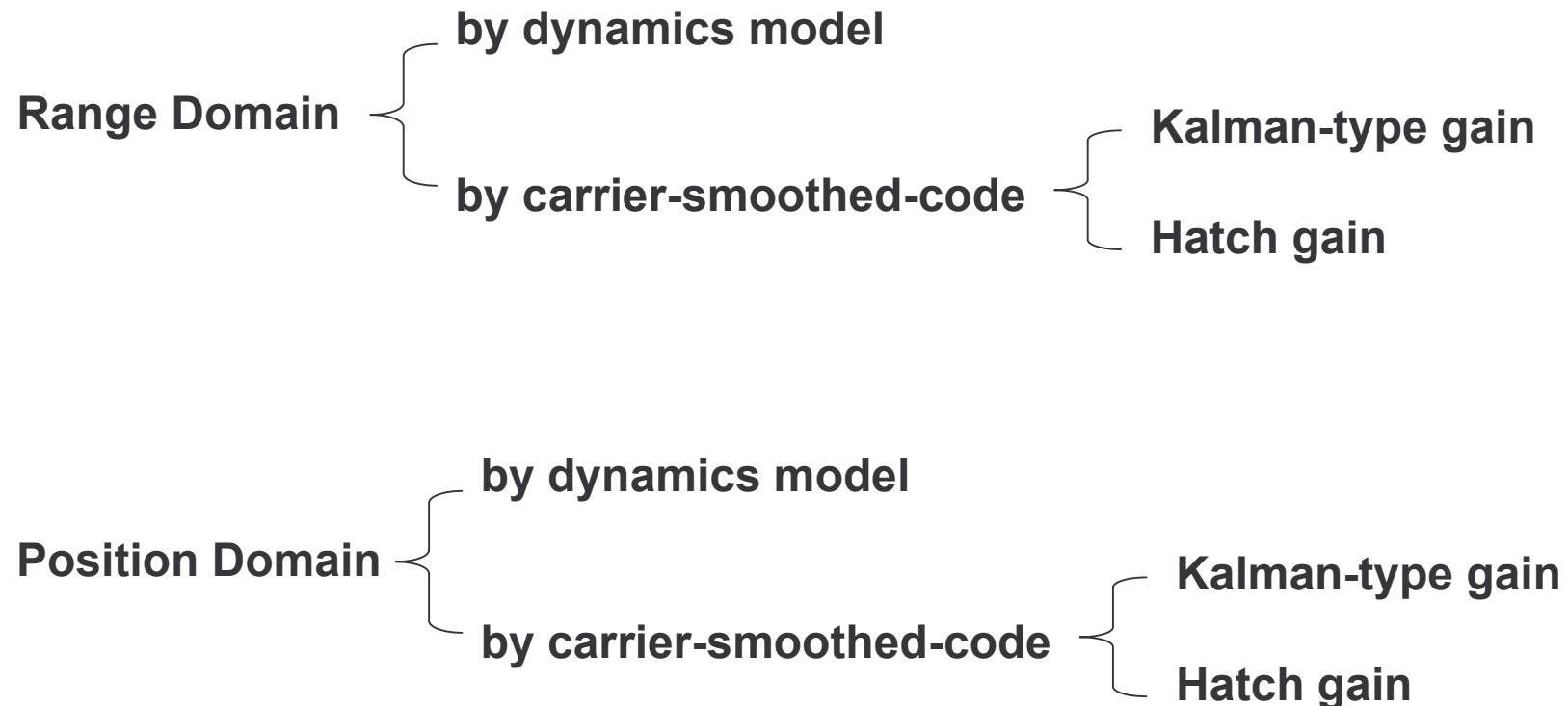
# Differential Filtering

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# *Classification of GPS-only Filters*



# **Types of Equivalent PRs**

\* Compressed PR (*a posteriori* PR estimate) for the j-th satellite

$$\hat{\rho}_{j,k} := E\left[\rho_{j,k} \mid \{\tilde{\rho}_{j,a}\}_{a=0,1,2,\dots,k}, \{\tilde{\phi}_{j,a}\}_{a=0,1,2,\dots,k}\right]$$

\* Projected PR (*a priori* PR estimate) for the j-th satellite

$$\bar{\rho}_{j,k} := E\left[\rho_{j,k} \mid \{\tilde{\rho}_{j,a}\}_{a=0,1,2,\dots,k-1}, \{\tilde{\phi}_{j,a}\}_{a=0,1,2,\dots,k}\right]$$

where

raw PR measurement:

$$\tilde{\rho}_{j,k} = e_{j,k}^T (x_{j,k} - x_{u,k}) + b_{u,k} + v_{j,k}$$

raw CP measurement:

$$\tilde{\phi}_{j,k} = e_{j,k}^T (x_{j,k} - x_{u,k}) + b_{u,k} + n_{j,k} + \lambda N_j$$

# *Carrier-Smoothed-Code Filters*

- \* Measurement model

$$\tilde{\rho}_{j,k} = e_{j,k}^T (x_{j,k} - x_{u,k}) + b_{u,k} + v_{j,k}, \quad v_{j,k} \sim (0, r_\rho)$$

$$\tilde{\phi}_{j,k} = e_{j,k}^T (x_{j,k} - x_{u,k}) + b_{u,k} + n_{j,k} + \lambda \mathbf{N}_j, \quad n_{j,k} \sim (0, r_\phi)$$

- \* Key idea: incremental range is directly measured  
dynamics model is not required

$$\begin{aligned}\tilde{\phi}_{j,k+1} &= \rho_{j,k+1} + n_{j,k+1} + \lambda \mathbf{N}_j \\ - ) \quad \tilde{\phi}_{j,k} &= \rho_{j,k} + n_{j,k} + \lambda \mathbf{N}_j\end{aligned}$$

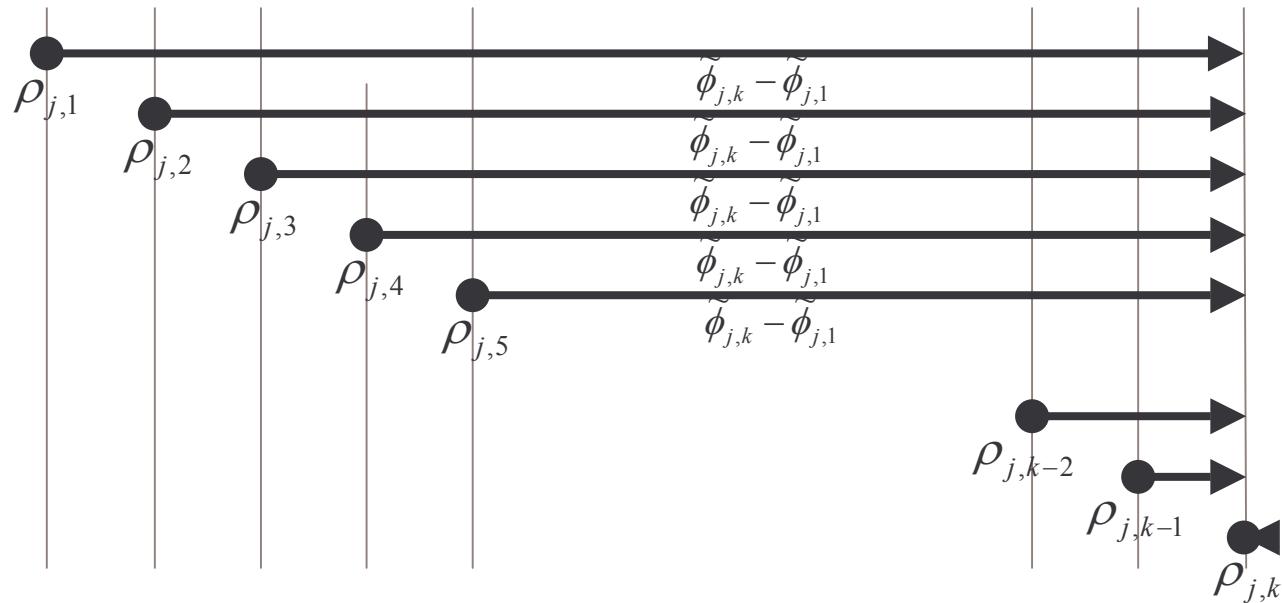
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$$\Delta \rho_{j,k} + (n_{j,k+1} - n_{j,k})$$

# Hatch Filter

## \* Principle

$$\begin{aligned}\hat{\rho}_{j,k} &= \frac{1}{k} \sum_{l=1}^k \left[ \tilde{\rho}_{j,l} + (\tilde{\phi}_{j,k} - \tilde{\phi}_{j,l}) \right] \\ &= \rho_{j,k} + \left[ \frac{1}{k} \sum_{l=1}^k v_{j,l} + \frac{1}{k} \sum_{l=1}^{k-1} n_{j,l} + \frac{k-1}{k} n_{j,k} \right]\end{aligned}$$



# *Positioning with Raw PRs*

$$\hat{X} = E[X | \tilde{\rho}]$$

## Code Example:

```
estusr = initpos;
while ((iter<maxiter)&(norm(beta)>tol)){
    for (N = 1;N<=numvis;N++){
        posdiff = svxyzmat.SubMatrix(1,3,N,N)-estusr.SubMatrix(1,3,1,1);
        pr0 = norm(posdiff);
        y(N,1) = prvec(N,1) - pr0 - estusr(4,1);
    }
    estusr_without_cb = estusr.SubMatrix(1,3,1,1);
    mt = H.t()*H;
    mt = mt.i();
    beta = mt*H.t()*y;
    estusr=estusr+beta;
    iter=iter+1;
}
```

# Positioning with Multiple RD Filters

\* Position Estimation by Compressed PRs

$$\bar{X}_k^r = E[X_k | \bar{\rho}_k], \quad \bar{X}_k^r \sim (O, \bar{P}_k^r), \quad \bar{P}_k^r = (H_k^T \bar{R}_k^{-1} H_k)^{-1}$$

\* Position Estimation by Projected PRs

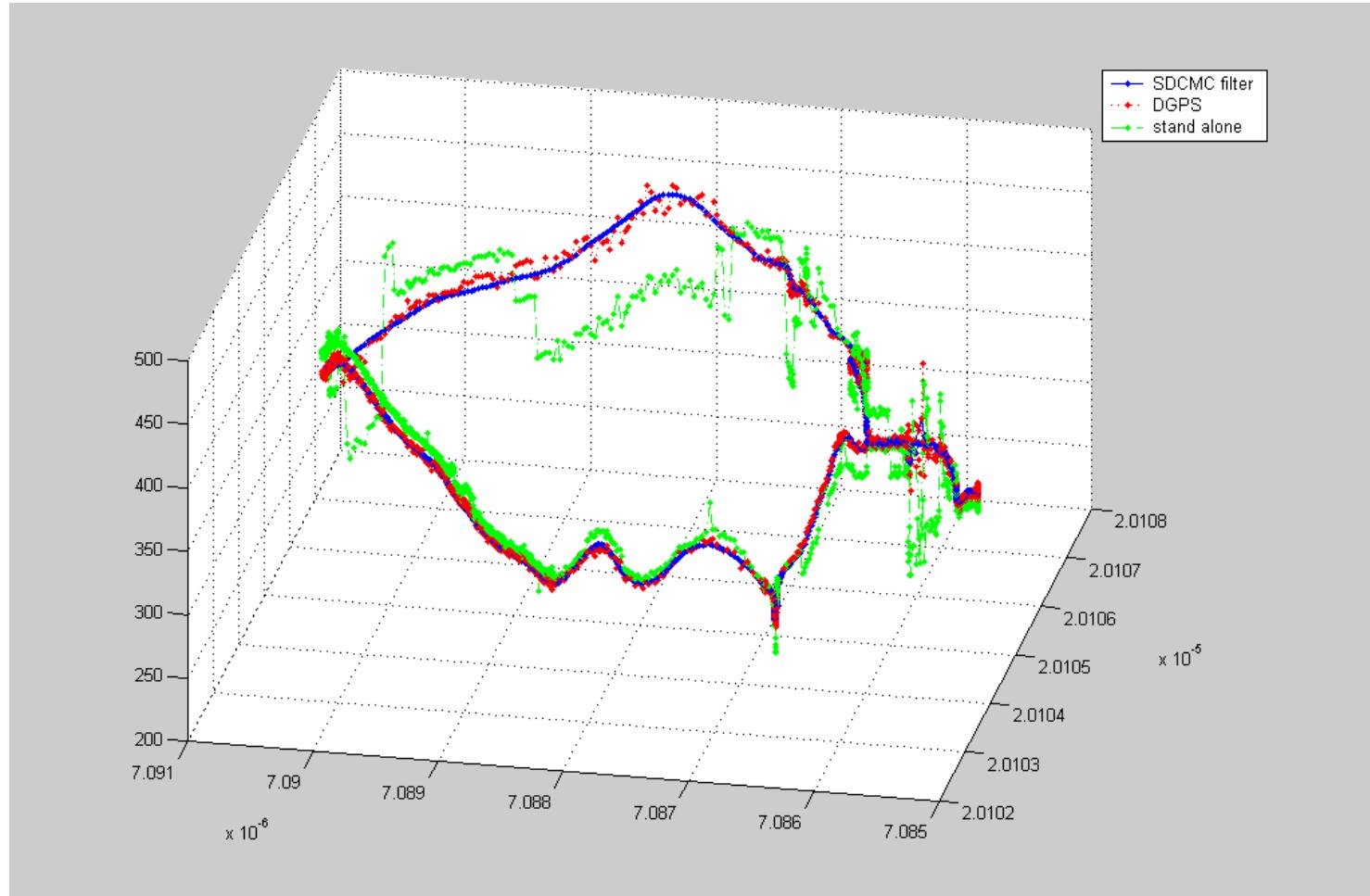
$$\hat{X}_k^r = E[X_k | \hat{\rho}_k], \quad \hat{X}_k^r \sim (O, \hat{P}_k^r), \quad \hat{P}_k^r = (H_k^T \hat{R}_k^{-1} H_k)^{-1}$$

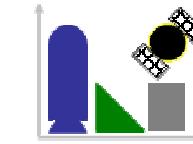
where

$$\bar{R}_k := \begin{bmatrix} \bar{R}_{1,k} & 0 & \cdots & 0 \\ 0 & \bar{R}_{2,k} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \bar{R}_{J,k} \end{bmatrix} \quad \hat{R}_k := \begin{bmatrix} \hat{R}_{1,k} & 0 & \cdots & 0 \\ 0 & \hat{R}_{2,k} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{R}_{J,k} \end{bmatrix}$$

Dimension of  $H_k$  and  $R_k$  varies according to the number of visible satellites  
→ If the signal from the satellite corresponding to the smallest  $R_k$  is blocked,  
large information loss occurs

# *Comparison: Standalone, DGPS, Filtered-DGPS*





# Carrier-phase Differential Positioning

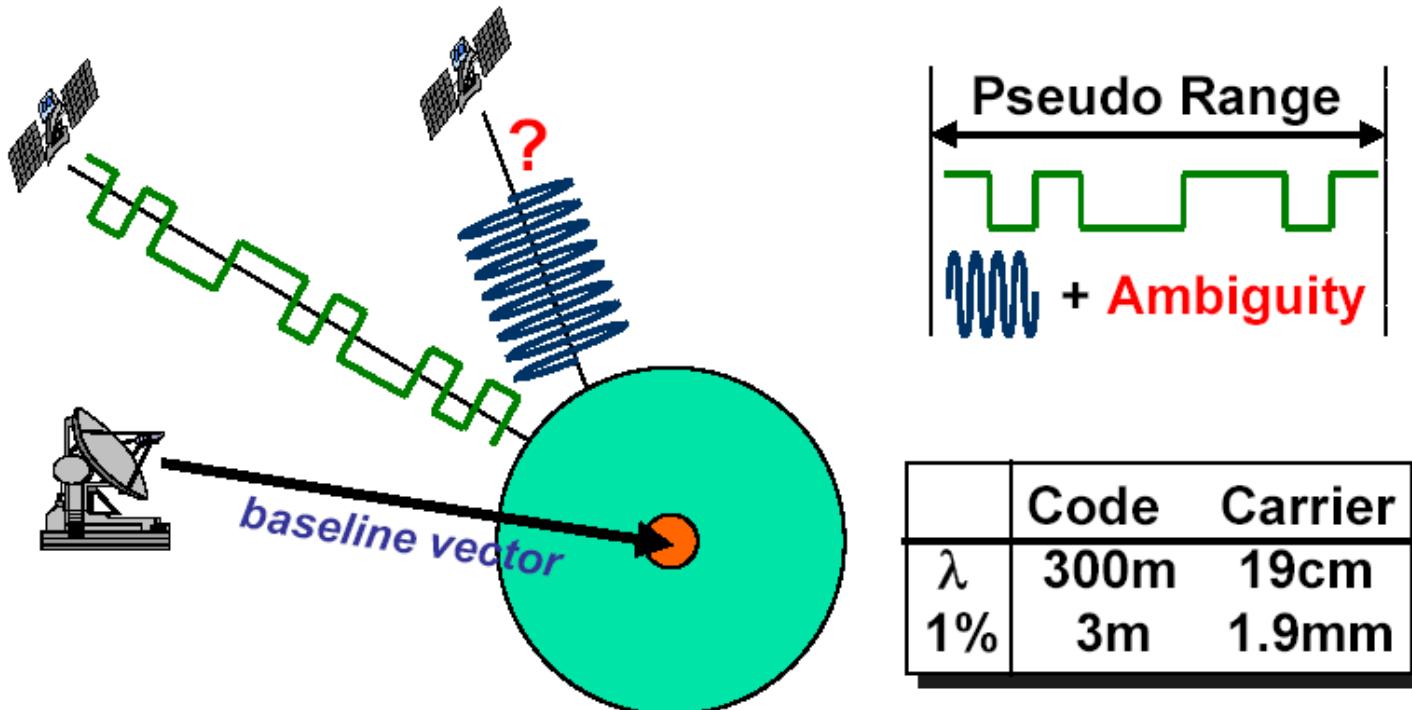
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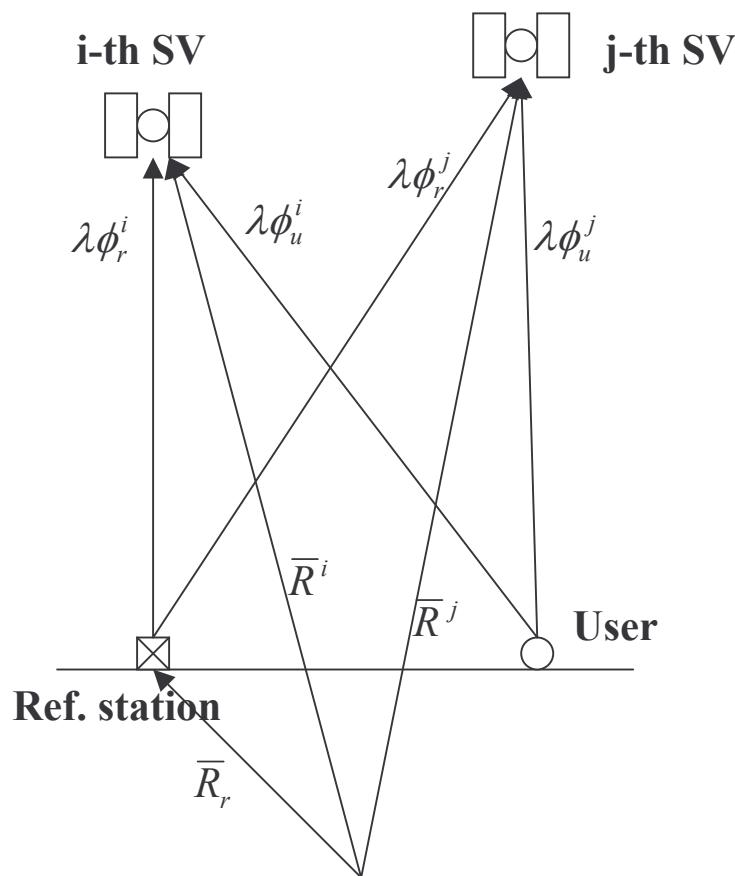
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# CDGPS 1

- CDGPS(Carrier phase Differential GPS)
  - GPS 위성이 송신하는 신호의 반송파를 이용하여 위치계산

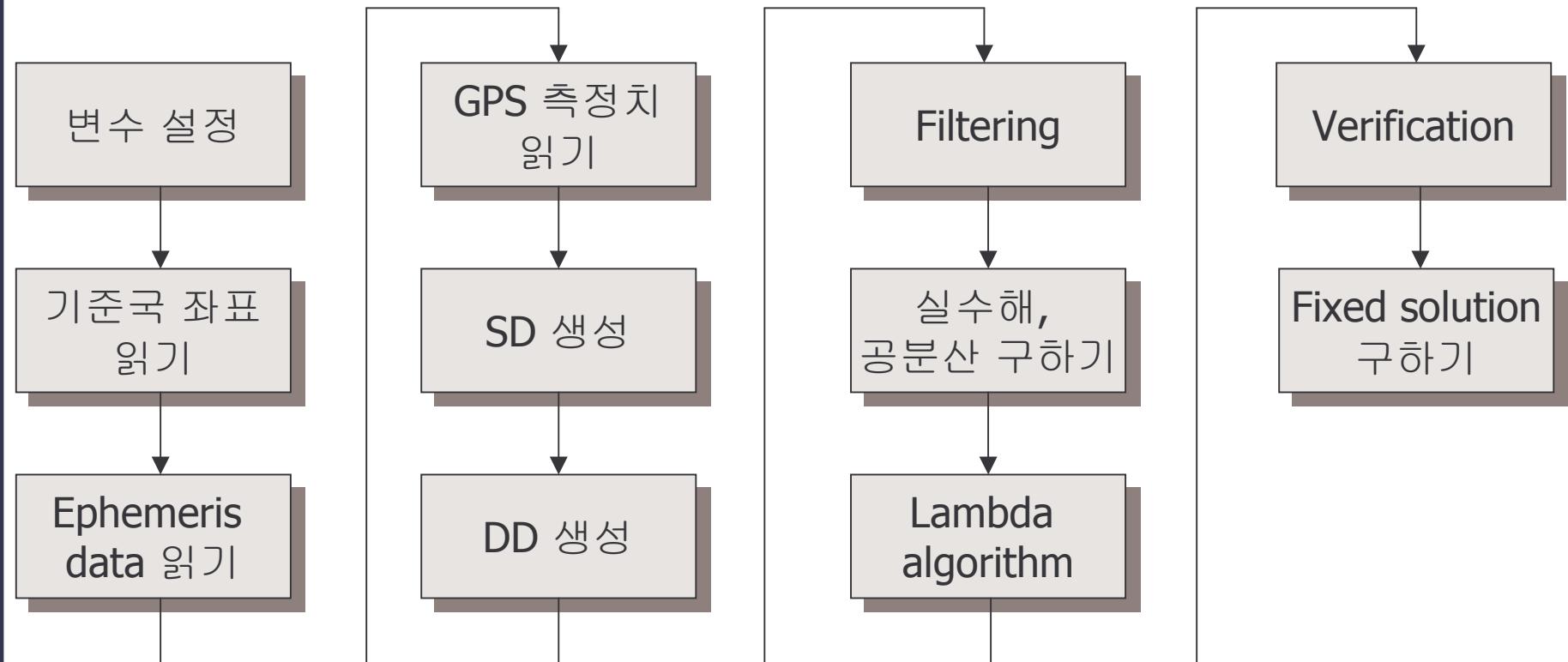




## ● 위치해 계산법

- 측정식 :  $\lambda\phi_r^i = d_r^i + B_r - b^i + \lambda N_r^i + \varepsilon_r^i$
- SD :  $\lambda\phi_{ru}^i = \phi_u^i - \phi_r^i = d_{ru}^i + B_{ru}^i + \lambda N_{ru}^i + \varepsilon_{ru}^i$   
DD :  $\lambda\phi_{ru}^{ij} = \phi_{ru}^j - \phi_{ru}^i = d_{ru}^{ij} + \lambda N_{ru}^{ij} + \varepsilon_{ru}^{ij}$
- 근사방정식  
 $\rightarrow \lambda\phi_{ru}^{ij} = d_{ru}^{ij} + \lambda N_{ru}^{ij}$
- 정확한  $d_{ru}^{ij}$ 를 알 수 있다면 실수  $\tilde{N}_{ru}^{ij}$ 를 계산할 수 있다.
- 람다 알고리듬과 실수  $\tilde{N}_{ru}^i$ 로부터 미지정수  $N_{ru}^{ij}$ 를 계산
- 선형근사방정식  
 $\rightarrow \lambda\phi_{ru}^{ij} - \lambda N_{ru}^{ij} = H_{ru}^{ij}[x, y, z]^T$
- 최소자승법에 의해 위치해 계산

# CDGPS 순서도



# Float Solution

- 다음과 같은 이중 차분식을 고려하면

$$y = Aa + Bb + e$$

여기서,  $y$  : 관측 데이터와 계산 데이터의 차분 벡터

$a$  : 이중차분 미지정수

$b$  : 기저선 벡터의 미지증분

$A, B$  : 행렬

$e$  : 오차

- **Float Solution(실수해)**

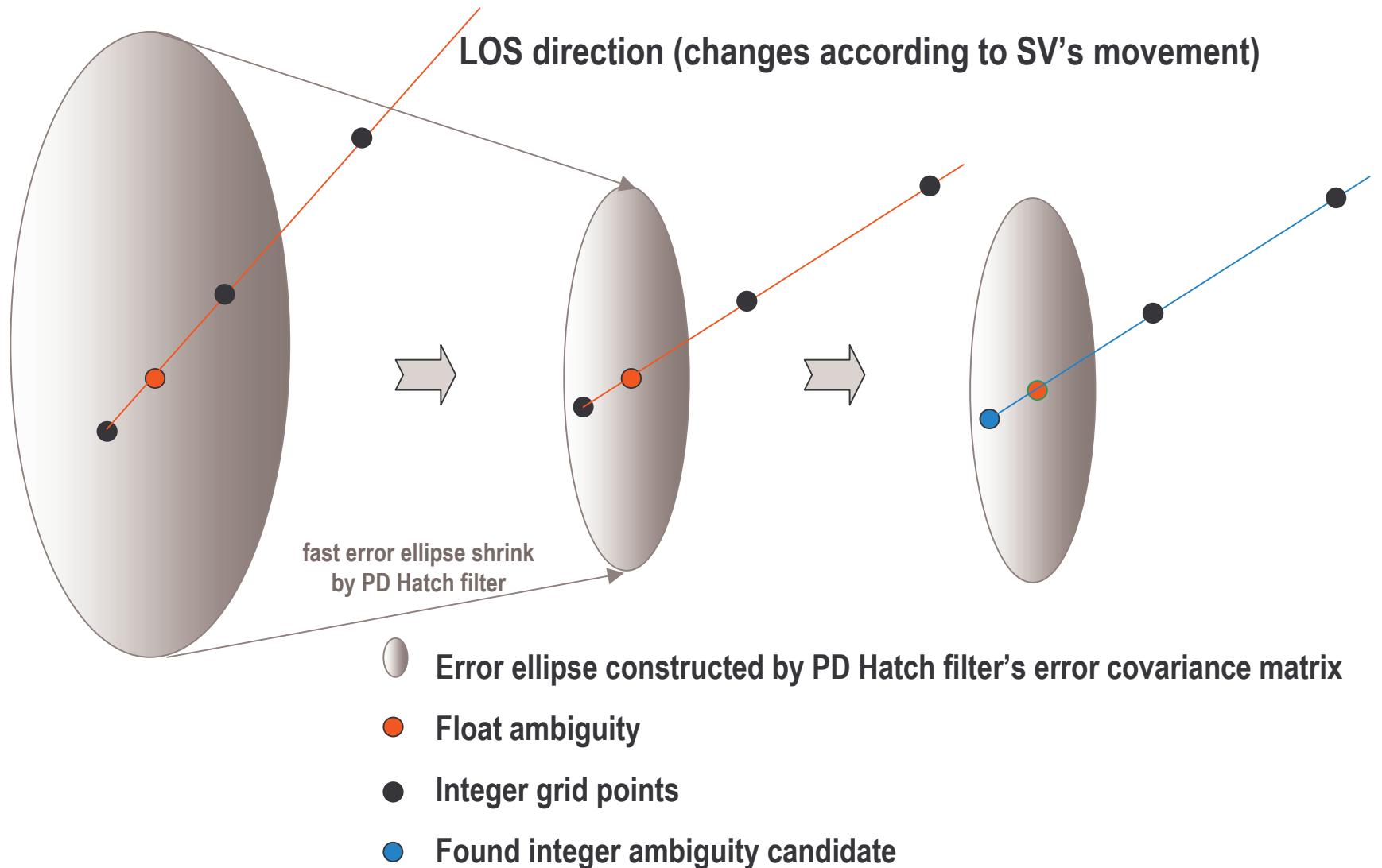
- 정수인 미지정수를 정수 조건을 제거한 후

- 다음의 Cost function를 가중 최소 자승법으로 푼 해

$$\min \|y - Aa - Bb\|_{Q_y}^2$$

where  $\|\cdot\|_{Q_y}^2 = (\cdot)^* Q_y^{-1}(\cdot)$   $Q_y$  : covariance matrix of  $y$

# *Selection of Integer Ambiguity Candidate*



# 미지정수 (*Integer Ambiguity*) 결정

1. Obtain  $\hat{\rho}$ ,  $\tilde{\Phi}$ , and  $R_{SMPR}$

$$DD \hat{\rho} = DD L X_{\text{baseline}} + DD \varepsilon$$

$$DD \tilde{\Phi} = DD L X_{\text{baseline}} + \lambda N + DD n$$

where

$DD$  : double - difference operator

$X_{\text{baseline}}$  : baseline from reference to user

$\hat{\rho}$  : smoothed pseudorange vector

$\tilde{\Phi}$  : measured carrier - phase vector

$\varepsilon \sim (O, R_{SMPR})$

$n \sim (O, R_{CR})$

$\sigma_{\min}(R_{SMPR}) \gg \sigma_{\max}(R_{CR})$

2. Subtract  $\hat{\rho}$  from  $\tilde{\Phi}$

$$DD \tilde{\Phi} - DD \hat{\rho} = \lambda N + DD n + DD \varepsilon$$

3. Compute  $\hat{N}$  and  $R_{\hat{N}}$

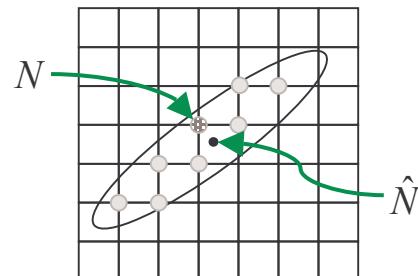
$$\hat{N} = \frac{1}{\lambda} (DD \tilde{\Phi} - DD \hat{\rho})$$

$$\hat{N} \sim (N, R_{\hat{N}}), R_{\hat{N}} \cong DD R_{SMPR} (DD)^T$$

4. Search each grid point

internal to the ellipse  $(\hat{N}, R_{\hat{N}})$

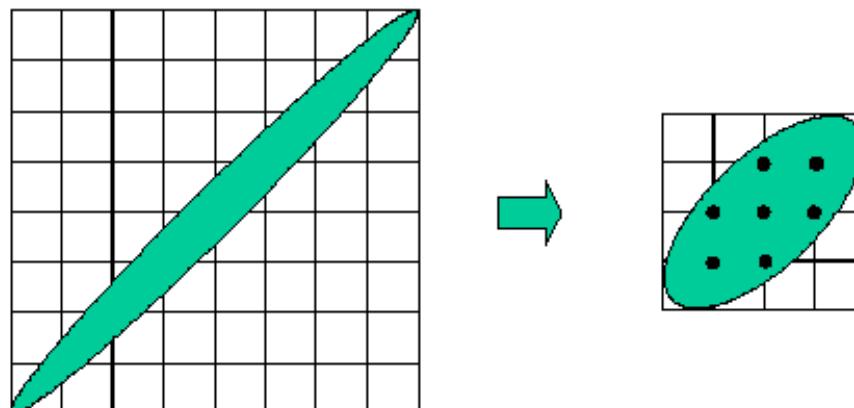
and compute distance from  $\hat{N}$



5. The minimum - distance point  
corresponds to the integer ambiguity  $N$

# LAMBDA 미지정수 결정 기법

- 이중 차분된 미지정수는 3개만이 독립되어 있음
  - Measurement 가 많을 경우 미지정수 간의 상관관계가 매우 커짐.
    - 불필요한 검색이 증가함
- 적절한 변환 행렬 Z 를 사용하여 미지정수의 상호 독립성을 높임
  - 기존의 미지정수 검색에서 발생하는 불필요한 검색을 줄임



미지정수 검색 공간의 효율화

# *Fixed Solution*

- 최소자승법으로 구해진 실수해와 공분산 행렬

- **실수해 :** 
$$\begin{bmatrix} B^T Q_y^{-1} B & B^T Q_y^{-1} A \\ A^T Q_y^{-1} B & A^T Q_y^{-1} A \end{bmatrix} \begin{bmatrix} \hat{b} \\ \hat{a} \end{bmatrix} = \begin{bmatrix} B^T Q_y^{-1} y \\ A^T Q_y^{-1} y \end{bmatrix}$$

- **공분산 :** 
$$\begin{bmatrix} Q_{\hat{b}} & Q_{\hat{b}\hat{a}} \\ Q_{\hat{a}\hat{b}} & Q_{\hat{a}} \end{bmatrix} = \begin{bmatrix} B^T Q_y^{-1} B & B^T Q_y^{-1} A \\ A^T Q_y^{-1} B & A^T Q_y^{-1} A \end{bmatrix}$$

- 목적함수를 최소화 하는 미지정수를 검색을 통해 결정

$$\min_a \| \hat{a} - a \|_{Q_{\hat{a}}^{-1}}^2 \quad \text{with } a \in Z^n$$

- 실수해와 공분산 2개의 입력을 이용
- 검색공간의 효율화를 위해 LAMBDA 방법을 사용

- **Fixed Solution**

- 결정된 미지정수를 다시 대입하여 정확한 위치해 계산

$$\breve{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} (\hat{a} - \breve{a})$$

# 미지정수의 검증

- LAMBDA 방법

- 미지정수의 검색을 보다 더 정확하게 효율적으로 하는 수단
  - 결정된 미지정수를 검증해야 잘못된 위치 해를 제거 가능

- 검증방법

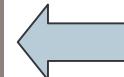
- 대부분의 경우 비율검사법을 이용

- 비율검사법

- 잔차의 weighted squared sum을 구하여 비율검사

$$\frac{\Omega_2}{\Omega_1} \geq \text{threshold ratio}$$

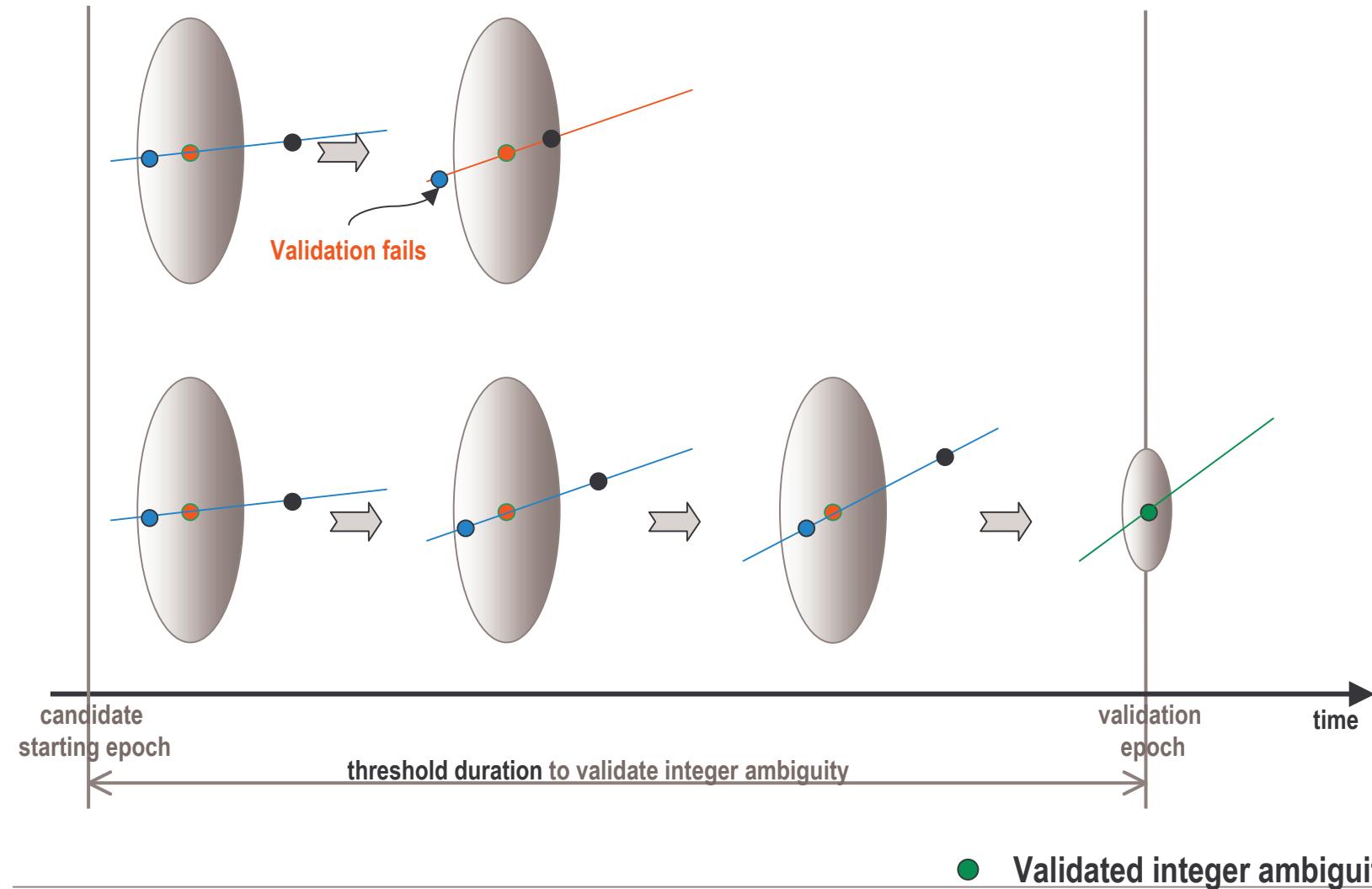
where,  $\Omega_1$  : smallest     $\Omega_2$  : one but smallest



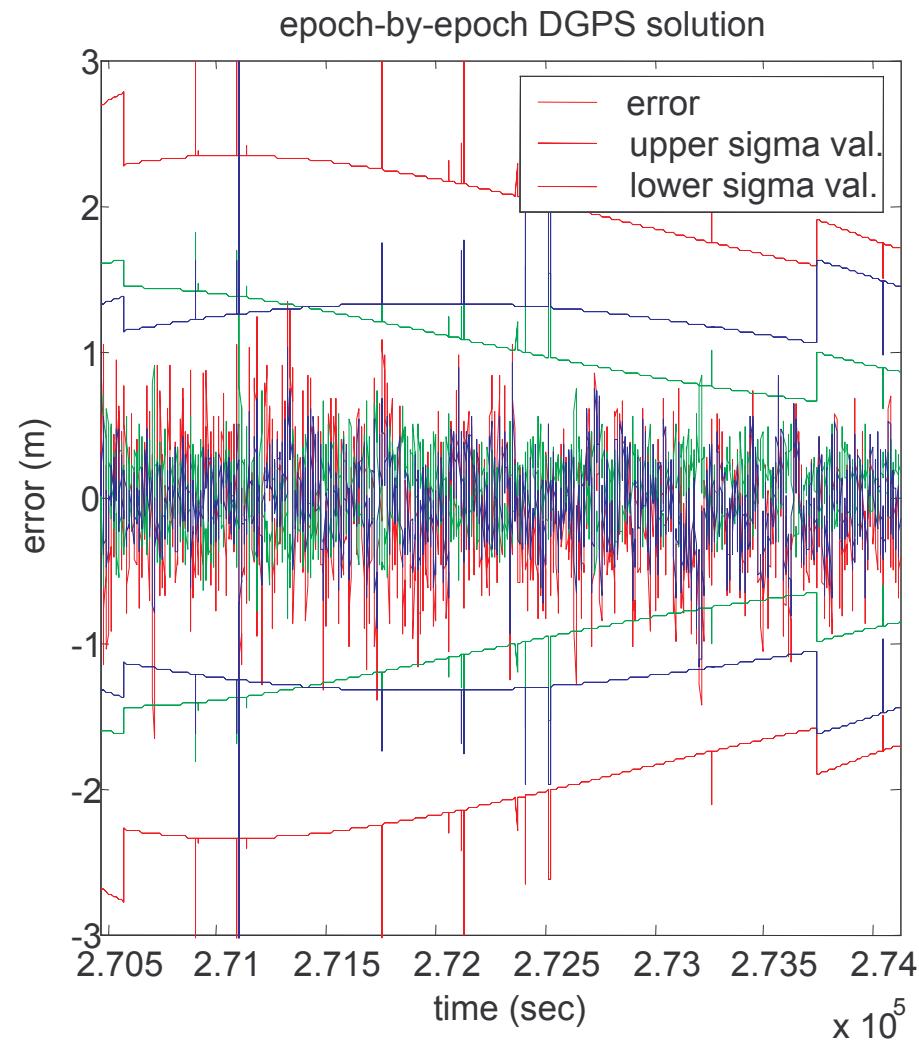
$$\Omega = (\hat{a} - \bar{a})^T Q_{\hat{a}}^{-1} (\hat{a} - \bar{a})$$

- 비율이 임계치[1.5-7사이의 값]을 넘는 경우 참 미지정수로 확정

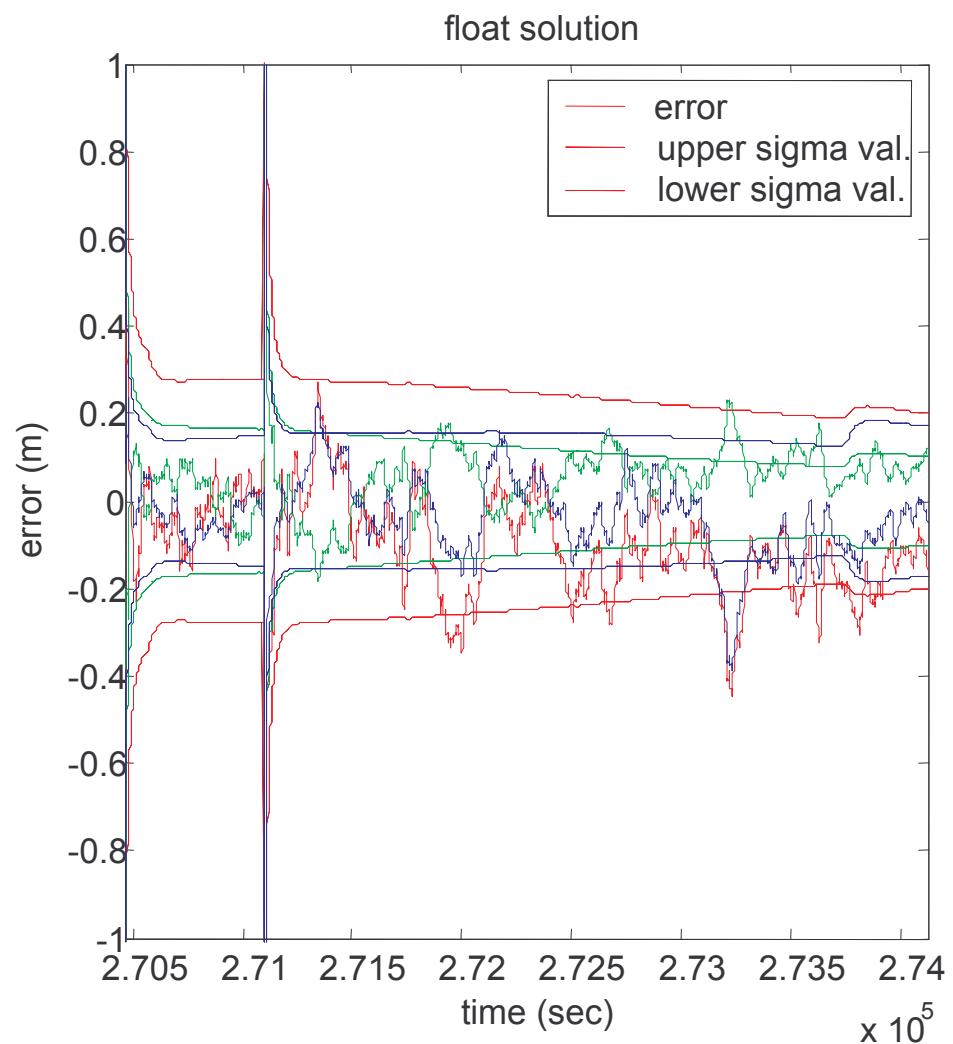
# *Validation of Integer Ambiguity Candidate*



# *Experiment: Epoch-by-Epoch DGPS Solution*



# *Experiment: Float Solution*



# *Experiment: Integer Solution*

